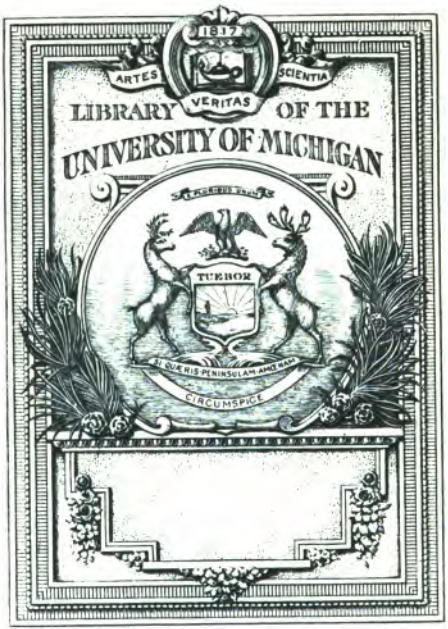
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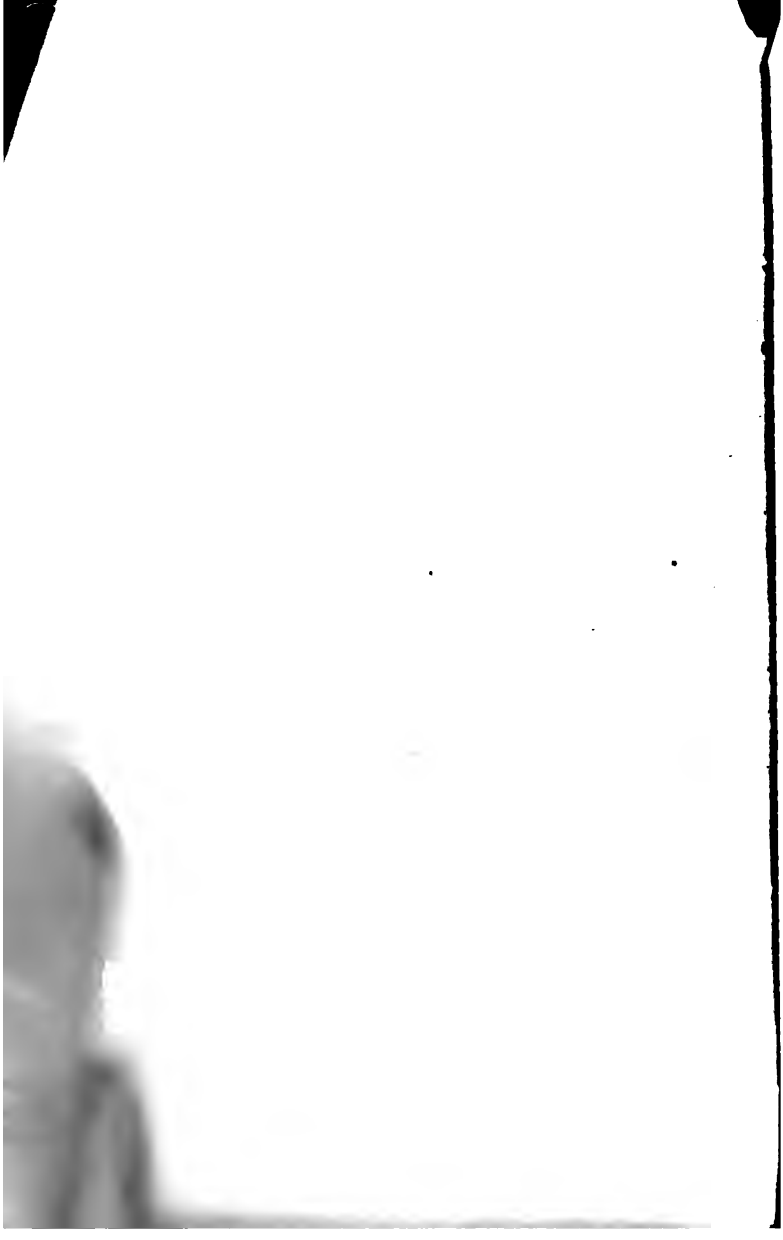

THE

PRUSSIAN CALCULATOR:

BY WHICH

ALL BUSINESS CALCULATIONS

ARE

PERFORMED BY ONE RULE.

WITH

AN APPENDIX.

BY I. A. CLARK,

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PREFACE.

During the last five or six years, the Author has made it his especial business to study and teach the science of numbers. In that time he has taught many, both of the young and old, and of both sexes.

In his classes he has seen the youth of five, and the silver locks of seventy; and as the author (with the favoring care of Providence) expects to give much of his time to this science as a teacher, he has compiled this work with that object in view.

It is true that the design of this work is to teach the principles of numbers used in the various business calculations of the day; yet, at the same time, it will invigorate and enlarge the mind; as it strips the science of Arithmetic of formula and rule, and causes the mind to rely on the great principles on which they are based. While the student stops behind the veil in many of the older systems, he is here invited to raise the curtain, enter the temple and view the interior in its simplicity and beauty.

The mind of the pupil is confused, his ideas indistinct, and his powers of analysis never exercised vigorously, while he depends on dead rules.

How often is the question put—To what rule does it belong? Or, Give me the rule and I can do it. Why should we not take nature for our guide? There we see but two principles, that of INCREASE and DECREASE; and the varied application of these, will solve every question that admits of solution in the science of numbers.

It is also true, that the given question always points to the mode of solution; and this is discoverable by analysis, which if followed will lead to a correct conclusion in every example.

I have often observed that the learner feels that with each step of advancement, a new principle was to be acquired, and that he was at each successive step learning a new principle,

and so learning a multiplicity of new things, not reflecting, that but two principles exist in regard to quantity as used in mathematics, and these as old as time itself.

In this treatise the author has confined himself to this principle, with the intention of developing a plain, concise, and practical method of solving business questions; such a method as the great mass of the people will use, as soon as they become acquainted with it. I speak with confidence, knowing that when this method is once thoroughly understood, its advantages are so great that it will never be exchanged for any other now in use; and this from the fact, that it is *plain, concise, uniform, direct* and *intelligible;* and will always be found CORRECT. It requires but a short time to acquire it, and experience cannot fail to recommend *one* mode for *many, one* rule (short and always applicable) in preference to *many,* in regard to some of which, there is often perplexity, doubt, and uncertainty, as to the result.

If, as I have stated, there are but two principles embraced in numbers—that of *increase* and *decrease*—why multiply rules? Let these be taken in their simplicity and investigated, and they cannot fail of leading to correct results.

It may be true that rules are of use; so are leading strings to the child, or floats to those who cannot swim; yet we had better rely on nature in either case. So it is with the science of numbers. When once we have learned the elementary principles, we are then prepared to be emancipated from technical rules, and should rely alone on general principles.

Indeed, we must pass by reliance on rules, before this science can enlarge the understanding or invigorate the mind; and one object of this work is, to guide beyond that point, and cause us to rely on our own judgment of principle.

It is impossible for any one to be well skilled in solving questions, who is confined to rules. RULES MAY BE MATERIALLY VARIED, BUT PRINCIPLES ARE IMMUTABLE.

THE AUTHOR.

THE

PRUSSIAN CALCULATOR.

PART I.

THE UNIVERSAL RULE,

IS

INCREASE AND DECREASE.

REMARKS.

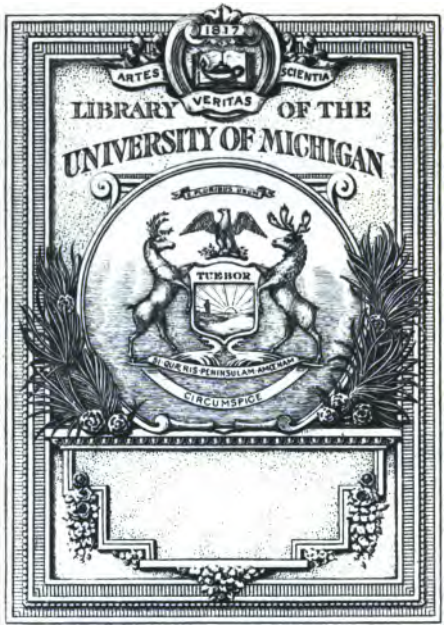
§ This rule is universal in its nature; and, variously applied, will solve every question in Mathematics that admits of solution.

§ In examining the various rules in the numerous Arithmetics now in use, we find that they all amount to the simple application of this principle.

§ The principle of Increase and Decrease is so clearly pointed out by nature in Mathematical problems, that it needs not a passing remark.

§ It is also true that the question presented for solution (if given in definite language) points to the mode of applying this principle in all cases.

§ Let the learner bear the above remarks in mind, and he may soon expect to be emancipated from all other rules.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7. Multiply 80 by 9, and that product by 21; and divide the whole product by 60 times 84.

2 ~~80~~	~~80~~ 4
3 ~~12~~ ~~84~~	~~9~~ 3
	21 3

3 Ans.

§ First cancel the ciphers, then divide 6 and 9 by 3. Then divide 8 by the 2 that stands on the left of the 6, cancel 8 and 2. Then divide 84 and 21 by 7, cancel both, and place the quotients (12 and 3) on the right and left of 84 and 21. Then say, 4 in 12 three times; cancel 4 and 12, and finally cancel the threes on opposite sides, and there will 3 remain on the right, which is the answer.

The following examples are given for the exercise of the learner:

Example 8.

2	1
4	3
3	4
6	5
5	6

$\frac{1}{2}$ Ans.

Example 9.

4	1
10	12
12	6
20	32
1	5

1 Ans.

Example 10.

20	5
30	24
36	45 9
12	10
1	27

9 Ans.

Example 11.

25	18
9	5
7	6
12	14

Example 12.

54	27
36	9
30	25
18	24

Example 13.

2	1
3	2
4	3
5	4

Example 14.

	40
12	9
30	20

Example 15.

100	800
12	6
30	15

Example 16.

12	840
30	7
	60

Example 17.

4	40
4	8
8	4

Example 18.

3	2
4	3
9	7
4	3
7	6
3	10
2	1
4	3
6	20
10	5

Example 19.

5	4
7	6
6	5
8	7
12	9
9	8
5	12
3	1
16	15
2	25

Example 20.

9	8
7	7
8	6
5	5
2	4
4	1
5	2
3	4
	1
	10

Example 21.

2	18
4	1
6	2
6	7
21	24
12	18
6	4

Example 22.

3	2
3	2
2	2
4	1
7	3
	9
	14

Example 23.

5	24
12	7
2	5
7	1
2	10
10	3
	25

Example 24.

3	2
9	7
4	3
7	3
3	6
2	1
4	24
6	16
20	30
25	27
	10

Example 25.

2	1
4	3
3	2
5	4
7	6
6	5
8	7
5	24
6	9
10	3
4	5

Example 26.

5	4
7	6
6	5
8	7
12	9
9	8
5	12
3	1
16	15 5
5	4
2	25

Example 27.

4	1
5	4
4	5
5	8
8	7
7	3
3	800
50	125
15	75
100	500
5	4

Example 28.	
9	63
49	35
50	70
42	36
18	27
45	20
16	28
14	26
39	21
56	64

Example 29.	
96	12
72	64
66	99
81	54
35	45
42	21
32	56
34	16
9	51
10	15

Example 30.	
75	15
80	50
72	96
84	42
50	60
70	100
63	49
45	27
54	90
8	144

Example 31.	
4	3
8	7
7	4
4	17
17	8
3	10
10	4

Example 32.	
4	3
8	7
7	4
2	7
7	6
5	24
12	7

Example 33.	
5	28
7	6
4	3
6	1
18	8
4	10
9	3

Example 34.	
40	60
4	8
48	40
10	4
34	17
22	11
2	20

Example 35.	
13	12
14	13
8	7
10	9
4	5
24	8
27	12

Example 36.	
2	1
5	4
2	7
7	1
4	2
12	5
3	8

Example 37.	
7	9
2	1
5	4
9	7
4	5
3	2
	3

Example 38.	
2	5
5	4
4	8
13	13
7	7
	6
	4

Example 39.	
3	1200
12	10
30	81

Example 40.	
4	80
40	80
60	12

Example 41.	
12	270
30	5
	24

Example 42.	
12	180
30	10
	20

FRACTIONS.

REDUCTION OF FRACTIONS.

To reduce a fraction to its lowest terms.

§ Place the numerator on the right, and the denominator on the left of the line.

§ Then divide by any number that will divide both without a remainder; thus continue to divide, till you can divide both no longer by any number greater than unity.

EXAMPLES.

1. Reduce $\frac{16}{20}$ to its lowest terms.

Divide both terms by 4, and we have $\frac{4}{5}$ for the answer.

5 20 | 16 4

$\frac{4}{5}$ Ans.

2. Reduce $\frac{28}{42}$; $\frac{50}{75}$; $\frac{96}{144}$; $\frac{128}{250}$; $\frac{512}{1536}$; to their lowest terms. Ans. $\frac{2}{3}$; $\frac{2}{3}$; $\frac{2}{3}$; $\frac{1}{2}$; $\frac{1}{3}$.

4. Reduce $\frac{48}{72}$ to its lowest terms. Ans. $\frac{2}{3}$.
5. Reduce $\frac{180}{150}$ to its lowest terms.
6. Reduce $\frac{70}{140}$ to its lowest terms.
7. Reduce $\frac{16}{192}$ to its lowest terms.
8. Reduce $\frac{288}{2652}$ to its lowest terms.
9. Reduce $\frac{1232}{3872}$ to its lowest terms.
10. Reduce $\frac{16}{24}$ of $\frac{48}{60}$ to its lowest terms.

24 | 16 4
15 60 | 48 2

15 | 8 = $\frac{8}{15}$ Ans.

11. Reduce $\frac{96}{108}$ of $\frac{54}{192}$ to its lowest terms. Ans. $\frac{1}{4}$.
12. Reduce $\frac{21}{35}$ of $\frac{74}{105}$ to its lowest terms. Ans. $\frac{2}{5}$.

MULTIPLICATION OF FRACTIONS.

§ Place the numerators, both of the multiplier and multiplicand, on the right, and the denominators of both on the left of the line, then proceed as directed in the general rule.

EXAMPLES.

1. Multiply $\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{8}{9}$ of $\frac{7}{8}$ and show the answer.

$$\begin{array}{c|c} 2 & 1 \\ 4 & 2 \\ 3 & 2 \\ 5 & 4 \\ 6 & 5 \\ 7 & 6 \\ 9 & 8 \\ 8 & 7 \end{array}$$

$\frac{1}{9}$ Ans.

2. Multiply $\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{10}$. Ans. $\frac{1}{8}$

3. Multiply $\frac{1}{3}$ of $\frac{2}{4}$ of $\frac{5}{6}$ by $\frac{6}{10}$ of $1\frac{6}{8}$. Ans. $\frac{1}{8}$

4. Multiply $\frac{1}{4}$ of $\frac{3}{7}$ of $\frac{7}{9}$ by $1\frac{3}{10}$ of $1\frac{9}{13}$.

5. Multiply $\frac{8}{9}$ of $\frac{3}{4}$ by $\frac{4}{9}$. Ans. $\frac{8}{9}$.

6. Multiply $\frac{6}{8}$ of $\frac{4}{12}$ by $\frac{7}{10}$ of $\frac{5}{14}$. Ans. $\frac{1}{16}$.

7. Multiply $\frac{7}{8}$ of $\frac{3}{14}$ of $\frac{8}{21}$ by $1\frac{4}{15}$ of $\frac{7}{6}$ of $\frac{5}{7}$. Ans. $\frac{1}{18}$.

8. Multiply $\frac{1}{3}$ of $\frac{3}{6}$ of $\frac{4}{8}$ of $\frac{6}{8}$ by $\frac{1}{2}$ of $\frac{4}{12}$. Ans. $\frac{1}{96}$.

9. Multiply $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{4}{5}$ by $\frac{6}{7}$ of $\frac{5}{9}$. Ans. $\frac{1}{35}$.

10. Multiply $\frac{3}{5}$ of $\frac{3}{4}$ of $\frac{4}{9}$ by $1\frac{6}{12}$ of $\frac{6}{15}$. Ans. $\frac{1}{15}$.

11. Multiply $\frac{1}{4}$ of $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{5}{9}$ of $\frac{5}{7}$ by $\frac{2}{5}$. Ans. $\frac{1}{9}$.

12. Multiply $\frac{1}{2}$ of $\frac{2}{4}$ by $\frac{6}{5}$ of $\frac{4}{13}$ of $\frac{3}{9}$. Ans. $\frac{1}{10}$.

CASE 2.

When it is required to multiply a whole number by a fraction, place the whole number on the right, and proceed as before.

EXAMPLES.

1. Multiply 16 by $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{1}{2}$.

$$\begin{array}{r|l} \cancel{4} & \cancel{16} \\ \cancel{5} & 1 \\ \cancel{2} & \cancel{4} \\ 2 & 1 \end{array}$$

1 Ans.

2. Multiply $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{4}{5}$ by $\frac{6}{10}$ of 10. Ans. $1\frac{1}{2}$.

3. Multiply $\frac{4}{5}$ of $\frac{1}{2}$ of $\frac{2}{7}$ of $\frac{14}{18}$ of $\frac{1}{4}$ by $\frac{6}{8}$ of $\frac{12}{13}$ of 26. Ans. $1\frac{4}{5}$.

4. Multiply $\frac{2}{4}$ of $\frac{4}{5}$ of $\frac{7}{12}$ of $\frac{6}{7}$ by $\frac{4}{8}$ of 5. Ans. $\frac{1}{2}$.

5. Multiply $\frac{3}{4}$ of $\frac{7}{8}$ by 16. Ans. $10\frac{1}{2}$.

6. Multiply $\frac{6}{7}$ of $\frac{9}{10}$ of $\frac{4}{12}$ by $\frac{1}{2}$ of 15. Ans. $1\frac{3}{14}$.

CASE 3.

§ When a whole number is joined to a fraction, reduce the whole number to the same denomination as the fraction with which it is joined, by multiplying the whole number by the denominator, and add in the numerator. Then proceed as before.

EXAMPLES.

1. Multiply $4\frac{1}{2}$ by $3\frac{1}{3}$.

$3\frac{1}{3}$ is equal to $\frac{10}{3}$, and $4\frac{1}{2}$ equals $\frac{9}{2}$.

$$\begin{array}{r|l} \cancel{3} & \cancel{10}\ \cancel{28} \\ \cancel{2} & \cancel{9} \\ 7 & \end{array}$$

14 Ans.

2. Multiply $\frac{1}{2}$ of $\frac{4}{5}$ of $3\frac{1}{2}$ by $\frac{1}{7}$. Ans. $\frac{2}{5}$.

3. Multiply $\frac{3}{4}$ of $\frac{7}{8}$ of $3\frac{1}{2}$ of $\frac{4}{7}$ of $\frac{3}{5}$ of $\frac{1}{2}$ by $\frac{3}{7}$. Ans. $\frac{1}{4}$.

4. Multiply $\frac{2}{6}$ of $\frac{4}{5}$ by $2\frac{1}{4}$. Ans. $\frac{3}{5}$.

5. Multiply $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{1}{6}$ by $5\frac{2}{5}$ of 5. Ans. 3.

6. Multiply $\frac{1}{3}$ of 9 by $\frac{4}{5}$ of 1. Ans. 3.

7. A lady having $\frac{4}{5}$ of a dollar, paid $\frac{1}{3}$ of her money for a comb; what part of a dollar has she spent?

Ans. $\frac{4}{15}$.

8. What will $2\frac{2}{3}$ lbs. of butter cost, at $8\frac{1}{4}$ cents per lb.? Ans. 22.

9. What will $6\frac{2}{3}$ lbs. of beef cost, at $2\frac{2}{3}$ cents per lb.? Ans. 16.

10. Required the cost of $\frac{1}{4}$ of $\frac{1}{6}$ of $\frac{3}{5}$ of a yard of cloth, at $\frac{1}{4}$ of $\frac{4}{5}$ of $5 per yard? Ans. $\$\frac{1}{25}$.

11. What will $3\frac{1}{2}$ acres of land cost, at $\frac{1}{5}$ of $\frac{2}{7}$ of $\frac{1}{4}$ of $50 per acre? Ans. $10.

12. I demand the cost of $\frac{1}{3}$ of $2\frac{1}{2}$ yards of cloth, at $\frac{2}{3}$ of $\frac{1}{2}$ of $5\frac{1}{4}$ per yard? Ans. $5\frac{1}{4}$.

13. What will $4\frac{1}{2}$ yards of satinet cost, at $\frac{2}{3}$ of $\frac{1}{12}$ of $\frac{6}{9}$ of $4 per yard? Ans. $6.

14. How many miles from New York to Philadelphia, it being $\frac{1}{2}$ of $\frac{1}{8}$ of $3\frac{1}{2}$ of $\frac{6}{10}$ of 480 miles? Ans. 90.

15. What is the value of 40 yards of cloth, at $37\frac{1}{2}$ cents per yard? Ans. $15.

$37\frac{1}{2}$ cents equals $\frac{3}{8}$ of a dollar.

$$\begin{array}{r|l} \cancel{40}\ 5 & \$ \\ \hline 3 & \$ \\ \hline 15 & \end{array}$$

16. What is the value of 48 yards of cloth, at $62\frac{1}{2}$ cents per yard? at $87\frac{1}{2}$? at $31\frac{1}{4}$? Ans. $30, $42, $15.

$62\frac{1}{2}$ cents equals $\frac{5}{8}$ of a dollar.

$$\begin{array}{r|l} \cancel{48}\ 6 & \$ \\ \hline 5 & \$ \\ \hline 30 & \end{array}$$

17. What is the cost of 64 yards, at $6\frac{1}{4}$ cents per yard? at $12\frac{1}{2}$? at $18\frac{3}{4}$? at 25? at $31\frac{1}{4}$? at $37\frac{1}{2}$? at $43\frac{3}{4}$? at 50? at $56\frac{1}{4}$? at $62\frac{1}{2}$? at $68\frac{3}{4}$? at 75? at $81\frac{1}{4}$? at $87\frac{1}{2}$? at $93\frac{3}{4}$?

18. What is the value of 66 yards of cloth, at $33\frac{1}{3}$ cents per yard? at $66\frac{2}{3}$? at 33?

19. What is the value of 600 yards of cloth, at 30 cents per yard? at 25? at 20? at 40? at 60? at 70? at 80?

20. What is the value of 12 yards, of cloth at 50 cents per yard? at 75? at $1,25? at $1,50? at $1,75? at $2,00? at $2,25? at $2,50? at $2,75? at $3,00? at $3,25?

DIVISION OF FRACTIONS.

§ Place the numerators of the divisor on the left and the denominators on the right, but place the dividend as in multiplication. If whole numbers are joined to a fraction, reduce as in multiplication.

PROBLEMS.

1. Divide $\frac{1}{8}$ of $\frac{5}{8}$ of $\frac{4}{7}$ by $\frac{7}{8}$ of $\frac{5}{12}$ of $\frac{8}{7}$.

$$\begin{array}{|c|c} \cancel{5} & 1 \\ \cancel{8} & \cancel{8} \\ 7 & \cancel{4} \\ \cancel{4} & \cancel{8} \\ \cancel{8} & \cancel{12} \\ \cancel{6} & \cancel{4} \end{array}$$

$\frac{1}{7}$ Ans.

2. Divide $\frac{1}{4}$ by $\frac{1}{2}$. Ans. $\frac{1}{2}$.
3. Divide $\frac{1}{2}$ by $\frac{1}{4}$. Ans. 2.
4. Divide $\frac{3}{5}$ by $\frac{1}{2}$. Ans. $1\frac{1}{2}$.
5. Divide $\frac{1}{2}$ by $\frac{3}{5}$. Ans. $\frac{5}{6}$.
6. Divide $\frac{4}{5}$ by $\frac{2}{3}$. Ans. $\frac{1}{2}$.
7. Divide $\frac{4}{5}$ by $\frac{4}{7}$. Ans. $1\frac{3}{4}$.
8. Divide $\frac{6}{7}$ of $\frac{6}{4}$ by $\frac{4}{5}$ of $\frac{3}{4}$. Ans. $1\frac{1}{2}$.
9. Divide $\frac{5}{6}$ of $\frac{6}{8}$ of $\frac{2}{5}$ by $\frac{3}{5}$ of $\frac{34}{35}$. Ans. $\frac{1}{2}$.
10. Divide $\frac{1}{4}$ of $\frac{1}{2}$ by $\frac{1}{8}$. Ans. 1.
11. Divide $\frac{1}{8}$ of $\frac{1}{2}$ by $\frac{1}{4}$. Ans. $\frac{1}{4}$.
12. Divide $\frac{1}{2}$ of $\frac{1}{8}$ by $\frac{1}{4}$ of $\frac{1}{4}$. Ans. 1.
13. Divide $\frac{5}{8}$ of $\frac{4}{5}$ by $\frac{2}{5}$ of 5. Ans. $\frac{3}{2}$.
14. Divide $\frac{1}{2}$ of $\frac{1}{4}$ by $\frac{1}{5}$ of 10. Ans. $\frac{1}{64}$.
15. Divide $\frac{4}{5}$ of $\frac{1}{4}$ by $\frac{4}{5}$ of 12. Ans. $\frac{1}{12}$.
16. Divide $\frac{1}{2}$ of 2 by $\frac{1}{4}$ of 4. Ans. 1.
17. Divide $\frac{1}{4}$ of 4 by $\frac{1}{8}$ of 8. Ans. 1.
18. Divide $1\frac{1}{2}$ by 4. Ans. $\frac{3}{8}$.
19. Divide $2\frac{1}{2}$ by $\frac{1}{2}$ of 5. Ans. 1.
20. Divide $\frac{1}{2}$ of 6 by $2\frac{1}{2}$ of 3. Ans. $\frac{1}{5}$.

MISCELLANEOUS EXAMPLES.

1. What is $\frac{3}{4}$ of 56 ? Ans. 24.
2. What is $\frac{1}{7}$ of 28 ? Ans. 16.
3. What is $\frac{2}{5}$ of 15 ? Ans. 6.
4. What is $\frac{5}{6}$ of 42 ? Ans. 35.
5. What is $\frac{2}{3}$ of 60 ? Ans. 40.
6. What is $\frac{3}{8}$ of 48 ? Ans. 18.
7. What is $\frac{1}{5}$ of 50 ? Ans. 10.
8. What is $\frac{3}{5}$ of 80 ? Ans. 48.
9. What is $\frac{4}{9}$ of 90 ? Ans. 40.
10. What is $\frac{1}{4}$ of 108 ? Ans. 27.
11. What is $\frac{5}{6}$ of 120 ? Ans. 100.
12. What is $\frac{4}{5}$ of 100 ? Ans. 80.
13. What is $\frac{4}{8}$ of 72 ? Ans. 36.
14. What is $\frac{6}{8}$ of 96 ? Ans. 72.

15. If 1 yard of cloth cost 27 cents, what will $\frac{2}{9}$ of a yard cost ? Ans. 6.

16. A has $\frac{2}{3}$ of $\frac{3}{4}$ of a ship, and B $\frac{3}{5}$ of $\frac{4}{5}$; which is the greater share, and how much ?

17. If a bushel of grass seed is worth 8 bushels of oats, what will $6\frac{5}{8}$ cost ? Ans. 53.

18. If a man drink $\frac{1}{8}$ of a gallon of beer in one day ; how much will he drink in 32 days ? Ans. 4.

19. If one quire of paper cost $\frac{3}{10}$ of a dollar ; what will 20 quires cost ? Ans. $6.

20. Suppose a fire engine to throw $4\frac{1}{2}$ barrels of water in one minute ; how much will it throw at the same rate in 40 minutes ?

21. A farmer sold $\frac{1}{4}$ of a ton of hay for $3\frac{1}{2}$; what is the price of a ton at the same rate ? Ans. $15 50.

22. How many times is $\frac{2}{3}$ contained in 16 ? Ans. 24.

23. How many pair of gloves can you buy for $18, at $\frac{3}{4}$ of a dollar per pair ? Ans. 24.

24. If a man can walk $\frac{3}{10}$ of a mile in 4 minutes, how far can he walk in 6 hours at the same rate ? Ans. 27.

25. How much corn will grow on 16 acres, each acre producing $34\frac{1}{8}$ bushels ? Ans. 554.

26. If a man can build one mile of wall in 350 days, how long will it take him to build $\frac{4}{7}$ of it? Ans. 200.

27. Suppose a farm to contain $48\frac{4}{12}$ acres; how much in $\frac{1}{2}\frac{2}{3}$ of it? Ans. $23\frac{1}{3}$.

28. Multiply $\frac{1}{2}\frac{0}{0}\frac{8}{5}$ by $\frac{5}{8}$. Ans. $\frac{8}{4}\frac{3}{1}$.

29. Divide $\frac{9}{2\,4}$ by $\frac{4}{12}$. Ans. $1\frac{1}{8}$.

30. If $2\frac{1}{2}$ bushels of apples fill a barrel, how many bushels will fill 8? Ans. '20.

31. Multiply $\frac{4}{5}$ of $\frac{5}{12}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{1}{1}\frac{6}{2}$, and divide the product by $\frac{3}{5}$ of $\frac{9}{1\,0}$ of $\frac{8}{9}$ of $\frac{6}{7}$ of $\frac{4}{12}$ of 1.

32. Multiply $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{6}{8}$ of $\frac{5}{0}$ of $4\frac{1}{2}$ of 16, and divide the product by $3\frac{1}{2}$ of $\frac{4}{7}$ of $\frac{6}{8}$ of $\frac{7}{9}$ of $\frac{2}{1\,0}$ of 20.

RATIO AND PROPORTION.

EXAMPLES.

1. If 4 yards of cloth cost $12, what will 7 yards cost at the same rate? Ans. $21.

$$\begin{array}{r|l} & 12\ 3 \\ 4 & 7 \\ \hline & 21 \end{array}$$

§ It is evident that if the number of yards had been twice as great, the cost would have been twice as much, or $2\times12=24$; and had it contained three times as much, the cost would have been three times $12=36$, &c.

2. If a horse travels 30 miles in 6 hours, how many miles will he travel in 11 hours at the same rate?

$$\begin{array}{r|l} & 30\ 5 \\ 6 & 11 \\ \hline & 55 \end{array}$$

Ans. 55 miles.

§ 6 hours are to 11 hours as 30 miles are to 55 miles.

§ Quantities which have the same ratio are said to be in proportion.

§ The ratio of two numbers is the quotient of the first divided by the second. Thus, in the first example, the ratio of 4 to 7 is $\frac{4}{7}$, and of 12 to the answer is $\frac{12}{21}$.

§ Ratio is usually expressed by two points written between the numbers; 4 : 7 or 12 : 21, which is read 4 is to 7 or 12 is to 21.

§ When two ratios are equal to each other, they may be written 4 : 7=12 : 21, which is read 4 is to 7 equals 12 is to 21, or they may be written thus; 4 : 7 :: 12 : 21, which is read 4 is to 7 as 12 is to 21.

§ Four numbers bearing such a relation to each other, are said to be in *proportion*.

§ The first term or terms in every ratio or proportion is called the antecedent, and the remaining terms the consequent. Thus 6 : 8=12 : 16; 6 and 12 are the *antecedents* and 8 and 16 are the *consequents*.

§ In every proportion the antecedents and consequents may exchange places. Thus, 3 : 4 :: 9 : 12 and 4 : 3 :: 12 : 9, or 9 : 12 :: 3 : 4, or 12 : 9 :: 4 : 3, are each true proportions.

§ The first and fourth terms of every proportion are called the extremes, and the second and third the means.

§ The proportion 4 is to 7 as 12 is to 21, or 4 : 7 :: 12: 21, may be written thus; $\frac{4}{7}=\frac{12}{21}$.

§ If these fractions were reduced to a common denominator, their numerators would be the same.

§ To find the numerators, multiply each numerator into the other denominator; then one numerator will represent the product of the means and the other the product of the extremes.

§ From this you will observe that the product of the means equals the product of the extremes.

§ Then when one extreme is given and the means; divide the product of the means by the given extreme, or place the given extreme on the left of the line and the two means on the right.

§ But if the two extremes are given and only one of the means, place the given mean on the left of the line and the two extremes on the right, and then proceed as in the general rule.

3. If 12 yards of cloth cost $6 what will 36 yards cost?

yds.		*yds.*		$		$
12	:	36	::	6	:	x

Ans. $18.

$$\begin{array}{r|l} 12 & 36\ 3 \\ x & 6 \\ \hline x & 18 \end{array}$$

§ As both the means are given in this example, they are both placed on the right, and the extreme that is given is placed on the left; then following the general rule we find the other extreme to be 18.

§ As the product of the means must equal the product of the extremes; for proof we place the answer on the left in place of the x, and then if the numbers will cancel, the answer is correct; but if they do not, then an error has been committed.

yds.		*yds.*		$		$
12	:	36	::	6	:	18

PROOF.

$$\begin{array}{rr|l} 12 & & 36\ 12 \\ 3 & 18 & 6 \end{array}$$

§ Read 12 yards are to 36 yards, as $6 are to $18, or as it was first stated, 12 yards are to 36 yards as $6 are to $$x$, that is, we do not yet know the number of dollars; therefore we place x in the blank term.

§ Say 6 in 18, 3 times, cancel 6 and 18, placing 3 (the times that 6 is contained in 18) on the left of 18; then find how many times 3 is contained in 36.

§ 3 in 36, 12 times, cancel 3 and 36, placing 12 (the times that 3 is in 36) on the right of 36. Then the twelves cancel.

§ The sum may be proven also by multiplying 36 by 6, $36\times6=216$, and 12 by 18, $12\times18=216$; and if their products are equal, the answer 18 is correct.

4. If 12 yards of cloth cost $6, what will $18 buy at the same rate?

yds.		*yds.*		$		$
12	:	x	::	6	:	18

$x \mid 12$
$6 \mid 18\ 3$
$x \mid 36$ Ans.

§ The blank now falls under one of the means; and to state the question, place the mean on the left; or if you use the blank (x,) place the means on the left.*

5. If 4 bushels of wheat cost $6, what will 8 bushels cost at the same rate? Ans. $12.

6. If 8 bushels of apples cost $12, what will 4 bushels cost at the same rate? Ans. $6.

7. If 4 bushels of apples cost $1,20, what will 60 bushels cost at the same rate? Ans. $18.

8. If 60 bushels of apples cost $18, what will 4 bushels cost at the same rate? 1 bush.? 8 bush.? 12 bush.? 16 bush.? 20 bush.? 40 bush.? Ans. $1,20; $0,30; $2,40; $3,60; $4,80; $6; $12.

9. If three lbs. of madder cost 27 cents, what will 8 lbs. cost at the same rate? Ans. 72 cents.

10. If 5 barrels of apples cost $35, what will 8 barrels cost at the same rate? 12 bbls? 17 bbls? 20 bbls? Ans. $56; $84; $119; $140.

11. If 5 yards of cloth cost $13,50, what will 75 yards cost at the same rate? Ans. $202,50.

12. If 2 lbs. of pork cost 8 cents, what will 1 lb. cost at the same rate? 40 lbs? 80 lbs? 120 lbs? 128 lbs? 150 lbs? Ans. 4c; $1,60; $3,60; $5,12; $6.

13. At 75 cents a bushel, what will 40 bushels of wheat cost? 60 bush? 75 bush? 100 bush? 120 bush? 130 bush? Ans. $30; $45; $56¼; $75; $90; $97½.

* The blank is always placed on the left, together with the term with which it is connected.

14. If a bushel of wheat cost 75 cents; what will 5 lbs cost at the same rate? 10 lbs? 15 lbs? 20 lbs? 18 lbs? 24 lbs? 30 lbs?

Ans. 6¼c; 12½c; 18¾c; 25c; 22½c; 30c; 37½c.

15. If 3 lbs of sugar cost 35 cents; what will 9 lbs cost at the same rate? Ans. $1,05.

16. If 4 yards of velvet cost $3, what will 16 yards cost at the same rate? Ans. $12.

17. What will one bushel of wheat cost if 420 bushels cost $336? Ans. 80c.

18. If 48 lbs of butter cost $6? what will 1 lb cost? 12 lbs? 14 lbs? 20 lbs? 23 lbs? 25 lbs? 32 lbs? 40 lbs? 41 lbs?

Ans. 12½; $1,50; $1,75; $2,50; $2,87½; $3,12½; $4,85; $5,12½.

19. If 72 yards of cloth cost $119,52; what will 9 yards cost? Ans. $14,69.

20. If a man spend 75 cents per day, how much is that a year? If he spend $1,25? $1,50? $1,75?

Ans. $273,75; $456,25; $547,50; $639,75.

§ *We find the following principles laid down in Philosophy, viz.:*

1. That like causes produce like effects.
2. That equal causes produce equal effects.
3. That effects are always in proportion to their causes.

§ Bearing the above premises in mind we apply the principle to all those causes and effects, the result of which can be computed by figures.

21. If 8 men in 12 days can perform a piece of work, how many will it require to perform a similar piece of work in 4 days?

1st cause.	*2d cause.*	*1st effect.*	*2d effect.*
8	x	1	1
12	4		

$$\begin{array}{c|l} x & 8 \\ 4 & 12\ 3 \\ 1 & 1 \end{array}$$

24

§ In the above example the first cause is 8 men 12 days, and the first effect is the work they do.

§ The second cause is part blank and 4 days, and the second effect is the work they do.

§ In all examples of this kind, the first cause and second effect are placed on the same side of the line, and the second cause and first effect on the opposite side.

§ The first cause and second effect are placed as the extremes, and the second cause and first effect as the means.

§ The blank is filled by x which shows that it is not known.

§ The blank, or x, is always placed on the left of the line, together with the term with which it is connected (i. e. the extreme or mean to which it is joined.)

22. If 12 men in 6 days can chop 72 cords, in how many days can 6 men chop 96 cords at the same rate?

1*st cause.*	2*d cause.*	1*st effect.*	2*d effect.*
12	6	72	96
6	x		

$$\begin{array}{rr|rr} & 6 & 12 & \\ & x & 6 & \\ \hline 6 & 72 & 96 & 16 \\ \hline & x & 16 & \end{array}$$

23. If 12 men in 6 days can chop 72 cords of wood; how many men in 16 days can chop 96 cords at the same rate? Ans 6. men.

1*st cause.*	2*d cause.*	1*st effect.*	2*d effect.*
12	x	72	96
6	16		

$$\begin{array}{rr|rr} & x & 12 & \\ & 16 & 6 & \\ \hline 6 & 72 & 96 & 6 \\ \hline & x & 6 & \text{Ans.} \end{array}$$

24. If 12 men in 6 days can chop 72 cords of wood, how many cords of wood can 6 men chop at the same rate in 16 days? Ans. 96 cords.

1st *cause.*	2d *cause.*	1st *effect.*	2d *effect.*
12	6	72	x
6	16		

$$\begin{array}{r|l} 12 & 6 \\ 6 & 16 \\ x & 72\ 6 \\ \hline x & 96 \end{array}$$

25. How many cords of wood can 12 men cut in 6 days, if 6 men can cut 96 cords in 16 days? Ans. 72 cords.

26. How many men will it require to cut 72 cords in 6 days, if six men can cut 96 cords in 16 days? Ans. 12 men.

27. In how many days will 12 men cut 72 cords of wood, if 6 men in 16 days cut 96 cords?

§ Note. Causes are those things that produce effects—as men at work, money lent, horses at work, time, &c.

§ Effects are the result of causes—as work performed, interest drawn, &c.

PROPORTION, OR SINGLE RULE OF THREE.

[CONTINUED.]

1. If 12 yards of cloth cost $6, what will 36 yards cost? Ans. $18.

2. If 4 bushels of wheat cost $6, what will 8 bushels cost? Ans. $12.

3. If 4 bushels of apples cost $1,20, what will 60 bushels cost? Ans. $18,

4. If 3 lbs. of madder cost 27 cents, what will 8 lbs. cost? Ans. 72 cts.

5. If 5 barrels of flour cost $35, what will 12 barrels cost? Ans. $84.

6. If 7 apples cost 28 cents, what will 20 cost at the same rate? Ans. 80 cts.

7. If 7 yards of cloth cost $40, what will 16 yards cost? Ans. $112.

8. If 6 horses eat 12 bushels of oats in a week, how many bushels will 30 eat in the same time? Ans. 60 bush.

9. If 5 horses eat 16 bushels of oats in 2 weeks, how long will it take them to eat 56 bushels at the same rate? Ans. 7 weeks.

10. If 3 lbs. of sugar cost 36 cents, what will 720 lbs. cost? Ans. $86,40.

11. If 8 yards of velvet cost $3,20, what will 48 yards cost? Ans. $19,20.

12. If 9 yards of cambric cost $14,93, what will 72 yards cost? Ans. $119,44.

EXAMPLES CONTINUED.

1. What will 421 bushels of wheat cost, if 4 bushels cost $2,88? Ans. $303,12.

2. What will 72 yards of cloth cost, if 9 yards cost $22,50? Ans. $180.

3. If 20 yards of superfine cloth cost $140, what will 7 yards cost? Ans. $49.

4. If 12 men can build a house in 48 days, in what time could 36 men build it? Ans. 16 days.

5. If 36 men can build a house in 16 days, how long would it take 12 men to build it? Ans. 48 days.

6. If 16 mowers can mow a field in 12 days, in what time can 24 men mow the same. Ans. 8 days.

7. How many yards that are 3 qrs. wide are equal to 30 yards that are 5 qrs. wide? Ans. 50 yards.

8. If 4 men can build a boat in 160 days, how long would it take 8 men to build it at the same rate? Ans. 80 days.

9. What will 16 yards of cloth cost if 8 yards cost $26? Ans. $52.

10. If 5 yards of cloth cost $30, how many yards may I have for $120? Ans. 20 yards.

11. What will 25 cords of wood cost if 3 cords cost $4,35? Ans. $36,25.

12. If 7 lbs. of sugar cost 56 cents, how much will $49 buy? Ans. 612½ lbs.

13. If 7 lbs. of coffee cost 70 cents, what will 18 lbs. cost at the same rate? Ans. $1,80.

14. If 12 inches in length and 12 in width make a square foot, how much in length that is 4½ inches broad will equal one square foot. Ans. 32 inches.

15. If a cent loaf weighs 12 oz. when wheat is 60 cents, what should be its weight when wheat is 40 cents?

Ans. 18 oz.

16. If I lend a friend $80 for 130 days, how long should he lend me $390 to requite my kindness? Ans. 26⅔ days.

17. If 64 soldiers eat 448 lbs. of beef in a week, how many pounds will 350 soldiers eat in the same time?

Ans. 2450 lbs.

18. If a staff 4 feet long casts a shadow 7 feet, what is the height of a tree that casts a shadow at the same time of 147 feet? Ans. 84 feet.

19. If a staff 5 feet 8 inches long casts a shadow of 6 feet, what is the height of a church spire that casts a shadow at the same time of 153 feet? Ans. 144½ feet.

EXAMPLES IN PROPORTION, HAVING MORE THAN THREE TERMS.

EXAMPLE 1. If 100 dollars gain $6 in 12 months, what is the interest of $400 for 8 months at the same rate?

Arrange the sum thus:

1st cause.	*2d cause.*	*1st effect.*	*2d effect.*
100	400	6	x
12	8		

100 | 400 2
12 | 6
x | 8

16 Ans.

2. If 8 men earn $4 in 3 days, in what time will 20 men earn $40? Ans. 12 days.

x	3
20	8 2
4	40 2
	12

3. If 10 horses eat 18 bushels of oats in 20 days, how many bushels will feed 50 horses for the same time?

Ans. 90 bushels.

4. If 10 horses eat 18 bu. of oats in 20 days, how many horses will 162 bu. feed for 36 days? Ans. 50 horses.

5. If 162 bushels will serve 50 horses 36 days, how many horses will 18 bushels serve for 20 days?

Ans. 10 horses.

6. If 16 men can mow 12 acres of grass in 3 days, how many men can mow 144 acres in 18 days?

Ans. 32 men.

7. If 4 men can mow 12 acres of grass in 3 days, how many acres can 8 men mow in 18 days? Ans. 144.

8. If 8 students spend 192 in 6 months, what will maintain 12 students 20 months at the same rate? Ans. $960.

9. If 8 men can do 24 rods of ditching in 6 days, how many rods may be done by 18 men in 24 days?

Ans. 216.

10. If 18 men in 30 days can build a wall that is 240 feet long, 8 feet wide, and 6 feet high, how long will it take 6 men to build one that is 80 feet long, 6 feet high, and 4 feet wide? Ans. 15 days.

11. If 6 men can build a wall 80 feet long, 6 feet wide, and 4 feet high in 15 days, in what time can 18 men build one 240 feet long, 8 feet wide, and 6 feet high?

Ans. 30 days.

12. If 4 men are paid 24 dollars for three days labor, how many men may be employed 16 days for $96?

Ans. 3 men.

13. If 7 men can mow 84 acres of grass in 12 days, how many men can mow 100 acres in 5 days?

Ans. 20 men.

14. If 40 cts. are paid for the carriage of 200 pounds, 40 miles, how far may 20200 pounds be carried for $60,60? Ans. 60 miles.

15. If 5 men spend $200 in 160 days, how long will $300 last 12 men at the same rate? Ans. 100 days.

16. If 12 oxen in 8 days eat 10 acres of grass, how many acres will serve 24 oxen 48 days? Ans. 120 acres.

17. A. has agreed to remove 8000 weight in 9 days, 15 miles; he moves 4500 with 18 horses in 6 days, how many horses must he employ to remove the balance in the remaining time? Ans. 28 horses.

18. How many pounds of thread will it require to make a piece of linen cloth 45 yards long, 4 quarters wide, if 4 pounds make 12 yards 5 quarters wide?

Ans. 12 pounds.

19. If a footman travels 240 miles in 12 days, when the days are 12 hours long, how many days will it take him at the same rate, to go 720 miles when the days are 16 hours long? Ans. 27 days.

20. Lent a friend 1200 dollars for 9 months; at the expiration of the time, received the interest, which was 54 dollars; at what rate per annum did I receive interest?

Ans. 6 per cent.

21. If 3 pounds of cotton make 10 yards of cloth 6 quarters wide, how many pounds will it take to make a piece 100 yards long, 3 quarters wide?

Ans. 15 pounds.

22. If 5 men can build 90 rods of wall in 6 days, how many rods can 20 men build in 18 days?

Ans. 1080 rods.

23. Being in want of money I procured 600 dollars of a broker for 8 months; at the end of that time I paid 48 dollars interest, what was the rate per cent. per annum?

Ans. 12 per cent.

24. If 5 men build 90 rods of wall in 6 days, how many rods can 12 men build in 15 days? Ans. 540 rods.

25. If 350 dollars in 9 months gain 15 dollars, what principal will gain 6 dollars in 6 months? Ans. $210.

26. How many weeks will 1440 dollars defray the expenses of 48 men, if 960 dollars defray the expenses of 20 men 88 weeks? Ans. 55 weeks.

27. If a family of 8 persons spend $480 in 24 months, how much will 16 persons spend in 8 months at the same rate? Ans. $320.

28. How many men can be employed 12 days for $96, if 4 men receive $24 for 6 days work? Ans. 8 men.

29. If $400 gain $24 in a year, what would $1600 gain in 20 months? Ans. $160.

30. If $400 gain $24 in a year, in what time would $1600 gain $160? Ans. 20 months.

31. If 45 yards of cloth, 6 qrs. wide, will make 10 suits of clothes, how many pieces each 45 yards in length, but only 3 qrs. wide, will it take to make 500 suits? Ans. 100 pieces.

32. If a wall can be built by 8 men in 12 days that is 40 feet long, 12 feet high, and 3 feet wide, in how many days can 68 men build another wall that is 28 feet high, 140 feet long, and 6 feet wide? Ans. $23\frac{1}{17}$.

33. If $2000 will support a garrison of 150 men 3 months, how long at the same rate will $6000 support 4 times as many men? Ans. $2\frac{1}{4}$ months.

34. If 25 men can dig a trench 36 feet long, 12 feet wide, in 9 days, in how many days would 15 men dig a trench of the same depth, 48 feet long and 8 feet wide? Ans. $13\frac{1}{2}$.

35. If a cellar which is $22\frac{1}{2}$ feet long, $17\frac{3}{10}$ feet wide, and $10\frac{1}{4}$ feet deep, be dug in $2\frac{1}{2}$ days by 6 men working $12\frac{3}{10}$ hours a day, how many days of $8\frac{1}{3}$ hours each, should 9 men take to dig another measuring 45 feet long, $34\frac{2}{3}$ feet wide, and $12\frac{3}{10}$ feet deep? Ans. 12 days.

36. If 15 men by working $6\frac{2}{3}$ hours per day, can dig a trench 48 feet long, 8 feet wide, and 3 feet deep in 12 days, how many hours a day must 25 men work to dig a trench 36 feet long, 12 feet wide, and 3 feet deep in 9 days? Ans. 6 hours.

47. Suppose that 50 men by working 45 days, each day 3 hours, can dig 24 cellars which are each 36 feet long, 21 feet wide, and 20 feet deep, how many men would be required to dig in 27 days 18 cellars which are each 48 feet long, 28 feet wide, and 15 feet deep in working only 5 hours each day? Ans. 50 men.

38. If 80 men by working 5 hours per day can dig in 27 days 20 cellars which are 45 feet long, 28 feet wide, and 10 feet deep, how many men would dig in 45 days 36 cellars which are 30 feet long, 21 feet wide, and 15 feet deep, by working only 3 hours per day?

Ans. 108 men.

INTEREST.

§ Place the principal, time and rate per cent. on the right. If the time consists of years and months, reduce the months to the aliquot parts of a year, or reduce the years to months, and add the months, placing their sum on the right, and 12, the number of months in a year, on the left. If the time consists of months and days, reduce the months to days, and place 12 and 30 on the left, or reduce the days to the aliquot part or parts of a month, placing only 12 on the left. Should the sum be large and you wish to cast the interest for days, place 365, or 73 and 5, on the left, and reduce the time to days.

PROBLEMS.

1. What is the interest of $12 at 6 per cent. for 9 months? Ans. 54 cents.

	12 Principal.
12	6 Rate.
	9 Time.
	54 Ans.

NOTE.—If you have the interest at 6 per cent, add one-sixth and you have the interest at 7 per cent, add one-third and you have the interest at 8 per cent.

2. What is the interest of $12 at 7 per cent.? at 8? at 9? at 10? at 11? at 12? at 13? at 14? at 15? at 16? for 9 months Ans. 63, 72, 81, 90, 99, $1,08, $1,17, $1,26, $1,35, $1,44.

3. What is the interest of $12 at 6 per cent., for 1 month? for 2? for 3? for 4? for 5? for 6? for 8? for 10? for 12? for 16? for 18? for 20? Ans. 6, 12, 18, 24, 30, 36, 48, 60, 72, 96, $1,08. $1,20.

4. What is the interest of $24 for 1 year and 2 months, at 6 per cent? Ans. $1,68.

	24 2
12	6
	14 months.

$1,68. Ans.

5. What is the interest of $24 for 1 year and 2 months, at 7 per cent? Ans. $1,96.

	24
12	7
	14

$1,96 Ans.

6. What is the interest of $48 for 5 days? for 15? for 20? for 45? at 6 per cent?

Ans. 4 cts., 12 cts., 16 cts., 36 cts.

	48
12	6
30	5

7. What is the interest of $48 for 6 months, at 6 per cent? Ans. $1,44.

8. What is the interest of $60 for 8 months at 6 per cent? Ans. $2,40.

9. What is the interest of $100 for 15 months at 6 per cent? Ans. $7,50.

10. What is the interest of $48,60 at 6 per cent for 1 year and 10 months? Ans. $5.35-.

11. What is the interest of $65,50 for 24 days at 6 per cent? Ans. $0,26,2.

12. Required the interest of $84,24 for 75 days, at 6 per cent? Ans. $1,05,3.

13. What is the interest of $106,80 for 9 months, at 3½ per cent? Ans. $2,67.

14. What is the interest of $720,12 for 1 year and 6 months, at 4⅘ per cent? Ans. $51,84,86.

15. What is the interest of $365 for 1 year and 3 months, at 12 per cent? Ans. $54,75.

16. What is the interest of $66,60 at 9 per cent. for 18 months? Ans. $8,99,1.

17. Required the interest on $8,40 for 2 years and 3 months, at 16 per cent? Ans. $3,02,4.

18. Required the interest on $99,99 for 1 year, 3 months, at 8⅓ per cent? Ans. $10,83.

19. What is the interest on $27,66 for 93 days, at 6 per cent? Ans. $,42,87.

20. Required the interest on $87,60 for 2 months, 15 days, at 7 per cent? Ans. $1,27½.

21. What is the interest on $366 for 1 year, 6 months, 20 days, at 6 per cent? Ans. $34,16.

22. Required the interest on $28 for 33 days, at 12 per cent? Ans. $,30,8.

23. Required the interest on $750 for 63 days, at 12 per cent? Ans. $15,75.

24. What is the interest of $264,18, for 4 months 20 days, at 30 per cent? Ans. $30,82,1.

25. What is the interest of 360 for 1 year, 2 months and 15 days, at 6 per cent? Ans. $26,10.

26. Required the interest of $8048 for 2 years 4 months, at 6 per cent? Ans. $1126,72.

27. What is the interest of $60,85 for 1 year 6 months, at 6 per cent? Ans. $5,47,65.

28. What is the interest of $60,96 for 1 year 8 months, at 6 per cent? Ans. $6,09,6.

29 What is the interest of $84 for 8 months, at 9 per cent? Ans. $5,04.

30. What is the interest of $800 for 1 year 3 months and 20 days, at 9 per cent? Ans. $94.

31. Required the interest of $375,50 for 1 year 10 months, at 6 per cent? Ans. $41,30½.

32. What is the interest on $76,40 for 1 year 5 months, at 8½ per cent? Ans. $9,19,98½.

33. What is the interest of $48,96 for 1 year, 4 months 12 days, at 6 per cent? Ans. $4,01,4+.

34. Required the interest of $180 for 8 months, at 6 per cent? Ans. $7,20.

35. What is the interest of $120 for 9 months, at 3½ per cent? Ans. $3,00.

36. Required the interest of $480, at 12 per cent., for 9 months? Ans. $43,20.

37. Required the interest of $99,99 for 5 months, at 4⅔ per cent? Ans. $1,94,42½.

38. What is the interest of $176, at 2¾ per cent. for 8 months? Ans. $3,22⅔.

39. What is the interest of $144 for 6 months 20 days, at 8 per cent.?

40. What is the interest of $77,64, at 2⅔ per cent. for 7 months 15 days? Ans. $1,16,46.

41. What is the interest of $2000 for 20 days, at 6 per cent?

42. What is the interest of $60 for 15 days, at 8 per cent?

43. Required the interest of $132 for 25 days, at 7 per cent? Ans. 64 cts.

44. What is the interest of $834 for 1 month 6 days, at 6 per cent? Ans. $5,00+.

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* In the New-England States, in New-Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, Tennessee, Kentucky, Ohio, Indiana, Illinois, Missouri, Arkansas, and the District of Columbia, and on all U. S. Notes, the rate is 6 per cent. In New-York, South Carolina, Michigan, Wisconsin, and Iowa, it is 7 per cent.

COMMISSION.

§ 1. Place the value of the commodity and the rate per cent. on the right, and place one hundred on the left.

EXAMPLES.

1. What is my commission on $8000 invested for A. in western lands, at 20 per cent. commission?

Ans. $1600.

2. What may I demand for the sale of 80 horses, valued at $75 per head, my commission being 12 per cent?

Ans. $720.

3. Sold 40 hogsheads of sugar on commission at 5 cents per cwt., each hogshead weighing 12 cwt., what is my commission? Ans. $24.

4. Sold on deposite, for C. & Co., 25 bales of goods, valued per bale at $400; what is my commission at 4½ per cent? Ans. $450.

5. C. sold on consignment $5000 worth of goods for B, what may he retain as his commission, it being 5 per cent?

Ans. $250.

6. Paid 4½ per cent. commission for the sale of $65400 worth of goods; how much was the commission?

Ans. $2943.

7. Sold 800 acres of western land; what is the amount of my commission at 15 per cent. per acre?

8. D. consigns to my care, goods to the amount of $25000, and allows me 9½ per cent. for making the sales. If I lay out $2375, how much of my commission have I left?

Ans. 0.

9. E. consigns to my care goods to the amount of $2500 when sold; how much must I send to E, my commission being 12½ per cent. Ans. $2187,50.

INSURANCE.

Insurance is a per centage paid for risk incurred in case of damage by fire, water, &c.

§ This is performed the same as the last.

EXAMPLES.

1. What is the insurance on a house valued at $40000, at $2\frac{1}{2}$ per cent? Ans. $1000.

2. What is the insurance on the steamboat Empire and cargo, valued at $140000, at $5\frac{1}{2}$ per cent? Ans. $7700.

3. What must I pay for the insurance of 4 houses, valued at $4500 each, at $3\frac{1}{2}$ per cent? Ans. $576.

4. If I own the $\frac{1}{4}$ of $\frac{1}{3}$ of $\frac{1}{3}$ of the steamboat United States, it being valued at 80000 dollars, what is my proportion of the insurance at $3\frac{1}{2}$ per cent? Ans. $44,44$\frac{4}{9}$.

5. The ship Julia Palmer was insured for $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $120000, at $4\frac{1}{2}$ per cent, what has A. to pay, he owning $\frac{1}{2}$ of $\frac{1}{12}$ of $\frac{6}{15}$ of the ship? Ans. $400.

6. What must C. pay for the insurance of his property, it being valued at $20000, at $2\frac{1}{2}$ per cent? Ans. $425.

7. If a ship is valued at $82500 and cargo at $25000, what is the insurance on the ship and cargo at $2\frac{1}{2}$ per cent? Ans. $2365.

8. What is the insurance on a mill valued at $100000, at $8\frac{1}{4}$ per cent? Ans. 8250.

9. What is the insurance on a cotton factory, at $2\frac{1}{2}$ per cent, valued at 40000? Ans. $1000.

10. What is the insurance on a woolen factory, at $2\frac{1}{2}$ per cent. valued at $50000? Ans. $1100.

DISCOUNT.

Discount is an allowance made for prompt payment.

CASE 1.

Discount without time.

§ 1. Place the sum on which the discount is to be made, and the rate per cent., on the right, and one hundred on the left.

EXAMPLES.

1. What is the discount on $400, at 6 per cent?
   Ans. $24.
2. What is the discount on $950, at 10 per cent?
   Ans. $95.
3. What is the discount on $300, at 8 per cent?
   Ans. $24.
4. What is the discount on $350, at $3\frac{1}{2}$ per cent?
   Ans. $12,25.
5. What is the discount on $1000, at 6 per cent?
   Ans. $60.
6. What is 12 per cent. of 700?
   Ans. $84.
7. What is 13 per cent. of $600?
   Ans. $78.
8. What is $3\frac{1}{3}$ per cent. on 1200?
   Ans. $40.
9. What is $4\frac{1}{3}$ per cent. of $500?
   Ans. $21.

BANK INTEREST,

OR DISCOUNT WITH TIME.

1. *To get the interest for months at six per cent.*

§ Place the principal and time in months on the right, and place 2 on the left.

2. *To get the interest for days at six per cent.*

§ Place the principal and time in days on the right, and place 60 on the left.

Three days are added to the time specified in notes taken by banks, called days of grace, on which interest is computed.

EXAMPLES.

1. What is the interest of $42 for 7 months, at 6 per cent? Ans. $1,47.

2	42 21 7

1,47

2. What is the interest of $120 for 33 days, at 6 per cent? Ans. $0,66.

60	120 2 33

66

3. What is the interest of $180,60 for 33 days, at 6 per cent? Ans. $0,99+.

60	180,60 301 33

,99,33.

4. What is the discount on $100 for 2 months, at 6 per cent. per annum? Ans. $1,05.

	100 5
12	6
30	03 21

5. What is the discount on $600 for 3 months, at 6 per cent. per annum? Ans. $9,30.

6. What is the discount on $120 for 4 months, at 12 per cent. per annum? Ans. $4,92.

7. What is the discount on $500 for 33 days, at 6 per cent. per annum? Ans. $2,75.

§ For any other rate than six per cent. compute the interest in the same manner, and add or subtract such parts as may be required, from 6 per cent. interest.

§ For three per cent. take one-half; for four per cent. subtract one-third; for four and a half per cent. subtract one-fourth; for five per cent, subtract one-sixth; for seven per cent, add one-sixth; for 8 per cent. add one-third; for 9 per cent. add one-half, &c.

§ This rule is the one generally used in banks and by business men.

§ It estimates the year at twelve months, of thirty days each; and consequently, gives the interest a trifle too large.

NOTE.—*The interest by the above rule, is $\frac{1}{73}$ too large, and to get the exact interest, deduct $\frac{1}{73}$ from the interest by the above rule.*

If you have only dollars in the principal, place the decimal point two figures from the right hand figure of the answer, and you have the answer then in dollars and cents.

If there are cents in the principal, place the decimal point four figures from the right, and you have the answer in dollars and cents, as before.

INTEREST ON NOTES.

To cast interest on notes where partial payments are made.

FIRST METHOD.

Cast the interest on the note from the time that it was given till the time of final settlement; then cast the interest on each payment from the time of payment till the final settlement; add these payments and interest together, and subtract the amount from the principal and the interest that has accrued on it.

SECOND METHOD.

The method laid down by CHANCELLOR KENT, is to apply the payment in the first place to the interest then due.

If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due.

If the payment be less than the interest, the surplus interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments taken together exceed the interest then due, and then the surplus goes towards discharging the debt.

The interest is computed on the balance as before.

LOSS AND GAIN.

To find the price at which to sell a commodity so as to gain or loose a given per cent.

§ Add the gain or subtract the loss per cent. from one hundred; and place it, together with the value of the commodity, on the right, and one hundred on the left.

EXAMPLES.

1. Bought cloth at 80 cents per yard. At what price must I sell it so as to gain 25 per cent? Ans. $1.

100	80 20
100	125

100 Ans.

2. Bought cloth at $1,00 per yard; at what price must I sell it so as to lose 25 per cent? Ans. 75 cts.

100	100
100	75

3. Bought cloth at $2,40; at how much per yard must I sell it so as to gain 20 per cent? Ans. $2,88.

4. What must I ask for 20 yards of cloth, that cost $2,-50 per yard, so as to gain 50 per cent? Ans. $75.

5. If 20 yards of cloth cost $75, at what price per yard must I sell it so as to lose 60 per cent? Ans. $1,50.

CASE 2.

When there is a gain or loss per cent, to ascertain what the commodity cost.

§ Add the gain or subtract the loss from one hundred, and place it on the left; and place one hundred, together with the price at which the commodity is sold, on the right.

EXAMPLES.

1. Sold a horse at $80 and lost 20 per cent; what did the horse cost? Ans. $100.

	$\$\emptyset$	100–20=80
$\$\emptyset$	100	

2. Sold a horse at $100, and gained 20 per cent; what did the horse cost? Ans. $\$83\frac{1}{3}$.

3. Sold cloth at $1,00 per yard, and gained 20 per cent; what did the cloth cost me? Ans. $83\frac{1}{3}$ cts.

CASE 3.

When a commodity is sold at a given rate, and there is a gain or loss per cent., to ascertain, what the gain or loss per cent. would be if sold at some other rate.

§ Place the first price and one hundred on the left, and having added the gain or subtracted the loss from one hundred, place this, together with the second price, on the right.

EXAMPLES.

1. If by selling cloth at 5 shillings per yard, I gain 10 per cent. what will I gain by selling the same for 6 shillings per yard? Ans. 32 per cent.

5	6	132
$10\emptyset$	$11\emptyset$	–100
1,32		32

2. If I buy cloth for $1,20 and sell the same for $1,50, what do I gain per cent? Ans. 25 per cent.

3. If I sell goods at 50 cts., that cost $62\frac{1}{2}$ cts., what is my loss per cent? Ans. 20 per cent.

EQUATION OF PAYMENTS.

§ 1. Place the sum or debt on the left, multiply each payment by the time that must elapse before it becomes due, and place the sum of these products on the right side of the line.

EXAMPLES.

1. C. owes $200, of which $100 is to be paid in 2 months and the balance in 4 months. If C. agrees to pay B. the whole sum in one payment, what is the equated time? Ans. 3 months.

+100	100×2
+100	100×4
2	6=3 months.

2. W. owes Craig & Co., $1300, of which 160 is to be paid in 6 months, $150 in 8 months, $400 in 9 months, and the balance in 10 months; what is the equated time? Ans. 9 m. nearly.

3. G. owes H. $100, of which $200 is to be paid down, $400 in 5 months, and the rest in 9 months, but they agree to one payment; what is the equated time? Ans. $5\frac{3}{7}$ m.

4. A farmer owes for a farm, and agres to make the payments as follows, or to pay the whole sum at the equated time : $\frac{1}{3}$ in 1 month, $\frac{1}{6}$ in 3 months, and the rest at the end of 12 months; what is the equated time? Ans. $6\frac{4}{9}$.

5. A merchant owes a bill of goods in Boston, valued at $6000, given January 1st, 1846, to be paid in four equal payments; the first in three months, the second in six months, &c. What is the equated time?

FELLOWSHIP.

§ 1. Place the whole amount of stock in trade on the left, and then place on the right the share of each partner severally, together with the whole gain or loss.

EXAMPLES.

1. Three men trading together gained $1600; A's stock was $1200, B's $2000, C's $4800, what was each man's share of the gain?

+1200	1600 2
+2000	1200+2000+4800.
+4800	

Or: A's $1200
B's $2000
C's $4800

$240 A's, $400 B's, $960 C's.

8000	1600 2
	1200+2000+4800

240 + 400 + 960.

2. E. F. and G. trading in company gained $240; E's stock was $280, F's $600 and G's $320; what was each man's share of the gain? Ans. E's $56.
F's $120.
G's $64.

3. Divide 360 into three parts that shall be to each other as 1, 2, and 3. Ans. 60, 120, 180.

4. Divide 720 into 4 parts that shall be to each other as 1, 2, 3, and 4. Ans. 72, 144, 216, 288.

5. Three boys found a purse containing $5,40, and finding no owner, agreed to divide it according to their ages; the youngest was 7 years old, the second, 9, and the oldest 11; how must they divide it? Ans. $1,40, $1,80, $2,20.

6. Three men pay for a pasture $120, into which A. puts 80 horses, B. 100, and C. 120; how much should each man pay? Ans. A. $32, B. $40, C. $48.

7. Three men agree to build a school house that will cost $600 and pay according to the value of their property. A. is worth $3000, B. $5000, and C. $7000; what must each man pay? Ans. A. $120, B. $200, C. $280.

8. A merchant failing in trade owes $24000; his whole amount of property is found to be worth only $6000: how much will B. receive to whom he owes $2000, or C. who has loaned him $3000.

Ans. B. receives $500 and C. $750.

9. How much will his property pay on the dollar?

Ans. 25 per cent.

NOTE.—By this rule tax lists are made out, bankrupts' estates are divided, and it will be found of use when there is a deficit of assets in various settlements.

When the shares of partners remain in trade unequal lengths of time.

§ Multiply each man's stock by the time that it remains in trade, then place the sum of the products on the left, and place on the right the product of each individual share, the whole gain or loss; then proceed as before.

EXAMPLES.

1. A. and B. trade in company; A's stock is $500 for 4 months, B's $600 for 6 months, and they gain $88; what the share of each. Ans. A's $31 3/7; B's $56 4/7.

500×4=2000
600×6=3600

7 $$$$ | 2000 5
$8 44
7 | 220=31 3/7

7 $$$$ | 3600 9
$8 44
7 | 396=56 4/7

2. A. B. and C. enter into partnership; A. puts in $85 for 8 months, B. puts in 60 for 10 months, and C. puts in $120 for 3 months; by misfortune they lose $41; what must each man sustain of the loss?

Ans. A. $17, B. $15, C. $9.

3. If A. should put in $215 and B. $390, and they should gain $200; what is each man's share, A's money being in trade 6 months, B's 9 months?

Ans. A's $53,75. B's $146,25.

4. E. F. and G. put in joint stock; E. puts in $144,00 for 5 years, F. $160 for $2\frac{1}{2}$ years, and G. $600 for 2 years, and they gained $720; what is each man's share of the gain?

Ans. E's $223,44$\frac{24}{29}$, F's $124,13$\frac{23}{29}$, G's $372,20$\frac{11}{29}$.

5. Two merchants join in trade, one puts in $4000 for 1 year, the other $3000 for 1 year and 4 months; they gain $1500; what is each man's share of the gains?

Ans. $750 each.

6. A. and B. charter a schooner and engage in trade. A. puts in $\frac{3}{4}$ of the stock and B. $\frac{1}{4}$; A. continues his sum in trade for 8 months, and B. his for 16 months, at the end of that time they close their accounts and find that they have gained $2400; what is each man's share of the gain if they divide in proportion to stock each had in trade?

Ans. A's $1440, B's $960.

7. Three merchants trade in company; A. puts in $240 for 10 months, B. $200 for 18 months, and C. $300 for 5 months; they gain $400; what is each man's share?

Ans. A's $128, B's $192, C's $80.

8. Three men form a partnership, with a capital of $12000. A's stock is $5000, for 8 months; B's $3000 for 12 months; and C's the balance for 16 months. They then have gained $2400. What is each man's share of the gain?

Ans. A's 685$\frac{5}{7}$.
B's 617$\frac{1}{7}$.
C's 1097$\frac{1}{7}$.

BARTER.

§ Place the given quantity of the commodity and the price at which it is valued, on the right of the line. Place on the left the constituents of the commodity whose value is required.

EXAMPLES.

1. How much cloth at 22 cents per yard, must be given in exchange for 4400 lbs. of cotton, at 3½ cents per pound?

Ans. 700 yards.

22	4400
2	7
	700

2. How much tea, at 64 cents perpound, must be given for 448 pounds of coffee, at 20 cents per pound?

Ans. 140 lbs.

3. How much wheat at $1,25 cents per bushel, must be given for 50 bushels of rye at 70 cts. per bushel?

Ans. 28 bush.

4. How many bushels of rye worth 70 cts. per bushel, must I give for 28 bushels of wheat, the wheat valued at $1,25 per bushel? Ans. 50 bush.

5. How many pounds of coffee can I have in exchange for 28 lbs. of butter, valued at 21 cts. per pound: the value of the coffee is 12 cts. per pound? Ans. 49.

6. How many sheep at $4 per head, must I give for 6 cows, at $12 a piece? Ans. 18.

7. Sold 28 bushels of wheat at 75 cts. per bushel; how many barrels of salt can I have in exchange at $2 per barrel? Ans. 10½.

8. How much coffee at 20 cts. per lb. must I give for 120 yards of cloth at 64 cts per yard? Ans. 384 lbs.

9. How many bushels of wheat will pay for 40 bbls. of pork at 8 dollars per bbl., when wheat is worth 80 cts. per bushel? Ans. 400 bu.

ALLIGATION.

Alligation teaches to find the value of mixtures.

§ 1. Find the value of the whole mixture, then find that of any part, by placing the value of the whole mixture and the quantity you wish to find the value of, on the right; and place on the left the whole amount of the mixture.

EXAMPLES.

1. If 4 bushels of wheat, worth 80 cents, be mixed with 8 that are worth only 60 cents, what is the value of 3 bushels of the mixture? Ans. $2.

$$\begin{array}{r|l} 12 & 4\times 80+ \\ & 8\times 60+ \\ & 3 \\ \hline & 200 \end{array}$$

2. If 6 gallons of wine at 67 cents per gallon, 7 at 80 cents, and 5 at $1,20 be mixed together, what is the value of 3 gallons of the mixture?

3. If 8 oz. of silver is melted with 16 oz., the first valued at 65 cents. per oz., the latter at 40, what is the value of 1 oz. of the alloy? Ans. 52$\frac{2}{3}$.

4. If I buy 40 bushels of rye at 60 cts., 30 bushels of corn at 50 cts., 60 bushels of barley at 25 cts., and have them mixed, what must I ask for 12$\frac{1}{2}$ bushels of the mixture? Ans. $4,61.

5. If I mix 40 bushels of corn at 50 cts., 28 of barley at 30 cts., and 22 of oats at 15 cts., what is 10 bushels of the mixture worth? Ans. $3,52$\frac{1}{2}$.

6. If 20 pounds of tea worth 50 cts., be mixed with 40 lbs. worth 80 cts., what is 10 lbs. of the mixture worth? Ans. $7.

THE

PRUSSIAN CALCULATOR.

PART II.

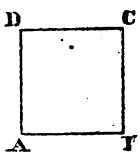
MENSURATION.

Practical Mensuration, or Practical Geometry.

There are three kinds of measures:

1. Linear or running, relating to the measure of lines.
2. Superficial or the measure of surfaces. This considers the length and breadth: i. e., the surface.
3. Cubic, or the measure of solids; this measure considers the length, breadth and thickness; i. e., the solid conents.

To find the area of a parallelogram, whether it be a square, a rectangle, a rhombus, or rhomboids.



Square.

§ Place the length of the base and the perpendicular height on the right, and on the left place the next inferior numbers that correspond with the answer.

EXAMPLES.

1. How many acres in a piece of ground that measure 160 rods on each side? Ans. 160 acres.

4 | 160
40 | 160

160

2. How many acres in a piece of land that measures 120 rods on each side? Ans. 90 acres.

3. What is the number of acres in a piece of land that measures 180 rods on each side? Ans. 202½ acres.

4. How many acres in a tract of land that measures 20 chains on each side? Ans. 40 acres.

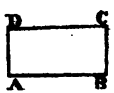
5. How many acres in a tract of land that measures 420 rods square? Ans. 1102½ acres.

6. What is the cost of a tract of land that measures 80 rods on each side, being a square, at $40 per acre?

Ans. $1600.

7. What is the cost of a square piece of land that measures 25 chains on each side, at $40 per acre?

Ans. $2500.



Parallelogram.

8. How many acres in a rectangular piece of land that measures 80 rods in length and 40 rods in breadth?

Ans. 20 acres.

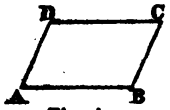
9. How many acres in a field that measures 40 rods in length, and 28 in width? Ans. 7 acres.

10. Required the cost of a rectangular piece of ground that measures 20 chains in length and 15 in width, at $25 per acre? Ans. $750.

11. How many acres in a lot that measures 400 rods in length and 200 in breadth? Ans. 500 acres.

12. Eight men purchase a tract of land, at $60 per

acre, which measures 40 chains in length, 48 in width, how much must each man pay? Ans. $1440.



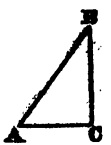
Rhombus.

13. I have a field in the form of a rhombus, the base of which measures 80 rods, the perpendicular measures 26 rods, required the area? Ans. 13 acres.

14. A field in the form of a rhombus measures 20 chains on either side, and a line from side to side measures 15 chains; how many acres in the field? Ans. 30 acres.

15. How many acres in a tract of land that is in the form of a rhomboid, measuring 40 chains on two opposite sides, and the perpendicular from one of these sides to the other, is 20 chains in length.

TRIANGLES.



Triangle.

To find the area of a Triangle.

§ Place the length of the base and the height on the right, and 2 and the next inferior denomination that corresponds with the answer, on the left.

EXAMPLES.

1. A triangular piece of board measures 3 feet across the base and 18 inches high; how many feet?

Ans. 2 feet 8 in.

$$\begin{array}{r|l} 4\ 12 & 3 \\ 2 & 12 \end{array}$$

2 ft. 3 in.

2. Required the area of a triangle whose base measures 180 rods, and perpendicular 80? Ans. 45 acres.

3. How many yards in the surface of a triangular wall that measures on the base 27 feet, and 25⅔ feet in perpendicular height? Ans. 38½ yds.

4. What is the value of a triangular piece of land at $60 per acre, that measures 60 rods at the base and 40 rods perpendicular height? Ans. $450.

5. What is the value of a triangular lot of ground that measures 200 feet on the base, 90 feet perpendicular height, at $30 per yard? Ans. $30000.

7. Required the area of a triangle whose base is 12¼ chains, perpendicular height 12 chains?

Ans. 7 ac., 1 rd., 16 rods, or 7,35 ac.

8. What is the area of an equilateral triangle whose base measures 20 chains, altitude 18 chains?

Ans. 18 acres.

9. What is the area of a scalene triangle; the base of which measures 198 rods, altitude 80? Ans. 49½ acres.

To measure Boards, Stone, Bark, Wood, Coal, &c.

§ Place the dimensions of the article to be measured, on the right; and the dimensions of any given quantity of the same, on the left.

EXAMPLES.

1. How many feet in 12 boards that measure each 16 feet long and 15 inches wide? Ans. 240 ft.

$$\begin{array}{r|l} 12 & 12 \\ & 16 \\ & 15 \end{array}$$

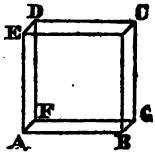
240

2. How many feet of boards that are 16 feet wide and 22½ feet long, are there in 8 tier, or 8 boards thick?

Ans. 2880 feet.

3. How many feet of scantling in 12 pieces, each 18 feet long and 4 inches square? Ans. 288 feet.

4. How many feet in 25 planks, each 20 feet long, 9 inches wide, and 2½ inches thick. Ans. 937½ ft.



A Cube,

has its length, breadth and thickness equal, and the solidity is found by multiplying these dimensions together.

5. How many cubic feet in a stick that is 40 feet long, 15 inches square? Ans. 62½ c. ft.

	40
4 12	15
4 12	15

62½ c. ft.

6. Required the cubic feet in a stick that measures 36 feet in length, and 10 by 12 inches thick? Ans. 30 ft.

7. Required the superficies of a board that measures 12 feet 6 inches in length, 14 inches in breadth?

Ans. 14 ft. 7 in.

8. How many feet in a board that measures 25 feet long, 12 inches at one end, and 18 at the other?

Ans. 31¼ feet.

+12	2	30 5
+18	12	25
30		31¼

9. If a log when sawed will make 10 boards, each 12 feet long, 15 inches broad, how many feet of boards?

Ans. 150 feet.

10. What will a pile of boards cost at $12 per 1000 ft., that measures 12 feet by 16 in the string, and containing 100 tier? Ans. $230,40.

11. If 4 feet high, 4 feet wide and 8 feet long, make one cord of wood, how many cords in a pile 40 feet long, 16 wide and 5 feet high? Ans. 25 cords.

$$\begin{array}{r|l} 8 & 40\ 5 \\ 4 & 16 \\ 4 & 5 \\ \hline & 25 \end{array}$$

12. How many cords in a pile that is 90 feet long, 12 feet wide and 8 feet high? Ans. 67½ cords.

13. How many cords of wood in a pile that measures 12 feet long, 16 wide, and 4 high? Ans. 6 cords.

14. How many cords in a pile that is 12 feet 6 inches long, 8 feet wide, 4 feet 6 inches high? Ans. 3½ cords.

15. Allowing 100 bushels of coal to measure the same as one cord of wood, how many bushels of coal will a box hold that is 8 feet long, 6 feet wide, and 5 feet deep?

Ans. 187½ bush.

16. How many bushels of coal can I put into a building of the following dimensions: 24 feet long, 20 wide, 16 feet deep? Ans. 6000 bush.

17. What will a lot of bark cost at $5,00 per cord, that measures 28 feet long, 20 feet wide and 5 feet high?

Ans. $109,37½.

18. I have purchased a lot of wood on board of a boat that measures 60 feet in length, 19 feet wide, 10 feet high, at $4,00 per cord; at what price must I sell the same to B., so as to gain 25 per cent. on the purchase?

Ans. $445,31¼.

16. How many bushels of coal will a box contain, that measures 4 feet wide, 10 feet long, and 3 feet 6 inches deep? Ans. 109⅜ bush.

MECHANICS' WORK.

CARPENTERS' AND JOINERS' WORK.

Flooring, Roofing, and Partitioning, are measured by the square.

§ A square is 10 by 10 feet, or 100 feet, and is to be placed on the left of the line.

EXAMPLES.

1. How many squares in a floor that measures 50 feet by 40? Ans. 20 sq.

2. How many squares in a floor that measures 25 feet by 20? Ans. 5 sq.

3. What will it cost to lay a floor of the following dimensions: 28 feet long, 25 feet wide, at $5 per square? Ans. $35.

4. What will it cost to lay 4 floors of the following dimensions: each 40 by 30 feet, at $2,50 per square? Ans. $120.

5. If a line passing over a roof from eave to eave, measures 60 feet, and the roof is 80 feet long, what is the cost at $2,50 per square for roofing? Ans. $120.

6. If a partition is 18 feet long and 10 high, how many squares? Ans. 1,8 sq.

7. If the walls round a room measure 100 feet in length and 8 feet in height, what is the cost at $5 per sq. for wainscoting? Ans. $40.

8. What is the cost of wainscoting a room at $1 per square yard, the compass of the room being 180 feet, height 12 feet? Ans. $240.

9. How many squares of weather boarding are on a house that measures 140 feet in compass, 20 feet high, making no deductions for openings? Ans. 28 sq.

MASONS' AND BRICK-LAYERS' WORK, &c.

Bricks are generally laid by the thousand. Deductions must be made for doors, windows, &c. Stone work is calculated by the perch, and 25 cubic feet are allowed in practice as a perch; but if the work demands accuracy, place 99 on the left and 4 on the right, in place of 25, it being $\frac{1}{100}$ too great. Painting is also done to some extent by the square yard.

To compute the number of bricks in a wall.

§ Reduce the brick to the fraction of a cubic foot, and place it on the left; and on the right place the dimensions of the wall.

EXAMPLES.

1. How many bricks in a wall that measures 50 feet in length, 12 feet high and 15 inches thick, each brick being 9 inches long, 4½ wide and 2½ thick?

Ans. 12,800.

	50
	12 4
9	15
9	2
5	2
	12 4
	12 4

12,800

§ As one of the dimensions is in inches, we place but two twelves on the right to reduce the wall to inches.

2. How many bricks in a wall that measures 180 feet compass, 18 feet high, and 18 inches thick, making $\frac{1}{5}$ deduction for the space occupied by mortar; the bricks of the same dimensions as those in the last problem?

Ans. 66,355.

NOTE.—In brick walls, one-fifth is sometimes deducted for the space occupied by the mortar.

3. How many bricks in the gables of a building that measures 30 feet in length, 12 feet perpendicular height, and 14 inches thick, brick same dimensions as before?
Ans. 7,168.

4. How many perch of stone in a wall that is 75 feet long, 7 feet high, and 3 feet thick; calling 25 cubic feet a perch? Ans. 63.

5. How many perch in a wall that is 180 feet long, 12 feet high, and 20 inches thick? Ans. 144.

6. How many perch in a wall that is 90 feet long, 18 feet high, 22 inches thick, estimating 24¾ cubic feet for a perch? Ans. 120.

7. How many yards of plastering in a room that measures 120 feet compass, 10 feet high, estimating only the walls? Ans. 133⅓.

8. What will the painting of a partition cost, at 12½ cts. per yard, that measures 28 feet long, and 12 feet high?
Ans. $4,66⅔.

9. How many yards of painting on a close fence, that is 150 feet long, 6 feet high? Ans. 100.

CIRCLES.

To find the area of a Circle, the diameter and circumference being given.

The diameter of a circle, is the line that passes through the centre from side to side.

The circumference, is the boundary line.

The radius of a circle, is half the diameter.

§ Place the diameter and circumference on the right, and 4, together with the next inferior number corresponding with the answer, on the left.

We divide by 4, for the reason that multiplying the diameter by the circumference, gives 4 times the area.

EXAMPLES.

1. What is the area of a circle that measures 280 rods diameter, and 880 rods circumference?

Ans. 385 acres.

4 | 280 35
4 | 880 11
40 |

385

2. What is the area of a circle that is 560 rods diameter, and 1680 rods circumference? Ans. 1470 acres.

3. What is the area of a circle whose diameter is 42, and circumference 132 rods? Ans. 8 ac. 2 roo. 32¼rd.

4. Required the area of a circle whose diameter is 40 chains, and circumference 125. Ans. 125 acres.

5. How many yards of pavement in a circle that is 120 feet in diameter, and 375 feet in circumference?

Ans. 1250.

6. In a circular floor that is 140 feet diameter, and 440 feet circumference; how many squares?

Ans. 154.

1. *To find the area of a Circle, the diameter being given.*

§ Place the square of the diameter, and 7854 on the right, and 10000 and the next inferior denomination, on the left.

2. *To find the circumference, the diameter being known.*

§ To get the circumference, multiply the diameter by 3,1416. The diameter being 1, the circumference is 3,1416, very nearly.

3. *To find the diameter, the circumference being known.*

§ To get the diameter, divide the circumference by 3,1416.

EXAMPLES.

1. What is the area of a field that measures 160 rods in diameter? Ans. 125,664 acres.

4	1600
40	1600
10000	7854

125,664

2. What is the area of a field that is a circle, the diameter of which is 40 rods? Ans. 7 ac, 3 roo, 16+rd.

3. Required the number of acres in a circular field, that is 120 rods in diameter? Ans. 70,7– acres.

4. What is the length of a tire that encircles a wheel of 4 feet diameter? Ans. 12 ft. 6,8 in.

5. What is the circumference of a wheel that is 8 feet in diameter? Ans. 25,13+ ft.

6. What is the area of a circle, the radius of which is 40 feet? Ans. 558,51 yds.

7. If 6 men purchase a circular farm that is 160 rods in diameter, at 40 dollars per acre, what has each to pay if they pay the same amount? Ans. $1060,29.

CYLINDERS.

To find the convex surface of a Cylinder.

§ Place the diameter, the number 31416, and height, on the right; and 10000 and the next inferior number that corresponds with the answer, on the left.

EXAMPLES.

1. What is the convex surface of a right cylinder, that measures 30 inches diameter, and length 24 feet?

Ans. 188,49+ ft.

12	30
10000	31416
	24

188,496

2. What is the convex surface of a right cylinder, that is 20 feet long, 24 inches diameter? Ans. 125,66+ ft.

12	24
10000	20
	31416

125,664

3. Required the superficies of a right cylinder, whose diameter is 30 inches, length 48 feet? Ans. 376,99+ ft.

4. What is the superficies of a shaft that measures 42 inches diameter, length 30 feet? Ans. 329,87– ft.

5. Required the convex surface of a well that is 54 inches diameter, 60 feet deep? Ans. 848,23+ ft.

To find the solidity of a Cylinder.

§ Place the square of the diameter, the decimal ,7854 and altitude, on the right; and then proceed as before.

EXAMPLES.

1. What is the solidity of a cylinder, the diameter of whose base is 24 inches, and length 30 feet?

Ans. 94,25– ft.

12	24
12	24
	30 5
	,7854

94,2480

2. Required the solidity of a cylinder, whose diameter is 12 inches, length 20 feet? Ans. 15,71– ft.

3. How many feet of timber in a stick that measures 9 inches in diameter, 40 feet in length? Ans. 17,67+ ft.

4. Required the solidity of a shaft that measures 48 feet in length, 20 inches diameter? Ans. 104,72 ft.

CONES.

To find the convex surface of a right cone.

§ Place the diameter of the base, the altitude and 31416, on the right; and 10000 and 2, the number corresponding with the answer, on the left.

EXAMPLES.

1.—On the fourth of July a pole was erected
Composed of six pieces and nicely connected,
Two feet six inches it measured around,
At the place where it stood at the top of the ground;
The form was a cone in surface complete,
The height of the same was twice sixty feet,
How many yards of inch ribbon procured at the shop
Will wind round this pole from bottom to top.
Laying smooth and plane to be seen
By leaving a space of five inches between?

Ans. 100 yds.

	60
2	2
3	30
6	1

100 yds.

2. The diameter of the base of a right cone is 90 inches, slant height 40 feet; what is the convex surface?

Ans. 471,24 ft.

3. The diameter at the base of a right none is 32 inches, slant height 48 feet, the convex surface is required?

Ans. 201,0624 ft.

4. Required the convex surface of a right cone, the diameter being 54 inches, slant height 36 feet.

	54 27
12	36
2	3,1416
	2544,696

5. What is the convex surface of a right cone, the diameter being 20 feet, slant height 30 feet 6 inches?

Ans. 958,19+.

To find the solidity of a Cone.

§ Place the square of the diameter at the base, the altitude and the decimal ,7854 on the right, and 3 and the number next inferior, on the left.

EXAMPLES.

1. What is the solidity of a cone, whose diameter at the base is 24 inches, altitude 20 feet?

Ans. 20,94+ ft.

12	24
12	24
3	20
	,7854
	20,9440

2. Required the solidity of a circular church spire, the diameter at the base being 12 feet, and height 60 feet?

Ans. 221,95+ ft., nearly.

3. The diameter of a cone is 20 feet, and its perpendicular height 24 feet. Required its solidity?

Ans. 2513,28 c. ft.

4. What is the solidity of a cone that measures 6 feet in diameter, slant height 36 feet. Ans. 94248 c. ft.

GLOBES.

To find the superficial area of a Globe.

§ If the diameter and circumference are both given, place the diameter and circumference both on the right, and the next inferior denomination on the left.

§ If the diameter only is given, place the square of the diameter, the decimal ,7854 and 4 on the right?

EXAMPLES.

1. What is the convex surface of a globe, the diameter being 8 inches, 25 inches circumference?

Ans. 1,4– ft.

3 12	8 2
6 12	25

1,3888

2. Required the area of a globe, whose diameter is 30 inches, circumference 94? Ans. $19\frac{1}{7}$.

3. The diameter of the earth is near 8000 miles, the circumference 25000; what is the area in square miles, of its surface? Ans. 200,000,000 miles.

To find the solidity of a Globe.

§ Place the square of the diameter, the decimal ,7854 and 4, together with one-sixth of the diameter, on the right; and place the next inferior denomination on the left, that corresponds with the answer; or place the cube of the diameter, and ,5236 on the right, and place the next inferior denomination that corresponds with the answer, on the left.

EXAMPLES.

1. What is the solidity of a globe that measures 24 inches in diameter? Ans. 4,1888 ft.

12	24
12	24
12	24
	,5236

4,1888

2. What is the solidity of a globe that is 12 feet in diameter? Ans. 904,7808 ft.

3. Required the solidity of the planet Jupiter, its diameter being 89,000 miles? Ans. 36921778400000.

4. Required the solidity of the planet Saturn, its diameter being 79,000 miles? Ans. 258156220400000.

5. Required the solidity of the earth in cubic miles, it being 8000 miles in diameter?

Ans. 268,083,200,000 c. miles.

GAUGING.

To find the number of gallons contained in casks of different forms.

§ Casks are divided into different classes, according to the curvature of their sides, and are calculated accordingly.

1st CLASS. Those that are cylinders.
2d " Those that are frustrum of a cone.
3d " Those that are two equal frustrums of a cone.
4th " The two equal frustrums of a parabola.
5th " The middle frustrum of a paraboloid.
6th " The middle frustrum of a spheroid.

NOTE.—Class first has no curvature, class 6 has the greatest.

CLASS 1.

§ Place the square of the diameter, the length, and 94 on the right, and if either of the dimensions are in inches, either

reduce them to the aliquot parts of a foot, or place 12 against inches, and 16 on the left.

EXAMPLES.

1. The diameter of a certain cistern is 8 feet, and height 10; how many gallons does it contain?

Ans. 3760.

	8 4
	8
16	10
	94

3760

2. How many gallons will a cylindrical cask hold, that measures 3 feet in height, and 16 inches in diameter?

Ans. 31,33+ gal.

CLASS 2.

§ Add the two diameters, and place the sum of their squares, the length, and 95, on the right; and place 4 and 16 on the left.

NOTE.—If either of the dimensions are in inches, reduce them to feet in all cases.

EXAMPLES.

1. Required the number of gallons a cask will hold, when the length is 3 feet, diameters 15 and 17 inches?

Ans. $31\frac{2}{3}$ gal.

12	
3 12	3
2	32
2	32 2
16	95

$31\frac{2}{3}$.

2. Required the contents of a cask, the length being 6 feet, diameters 30 and 34 inches at the ends?

Ans. $253\frac{1}{3}$ gal.

CLASS 3.

§ Place the dimensions as last directed, with 95 on the right and 16 on the left.

EXAMPLES.

1. A cask measures 15 inches end diameter, and 17 bung diameter, length 3 feet; how many gallons will it hold?

Ans. 31.66 gal.

12	
3 12	3
2	32
2	32
16	95

31⅝

2. Required the contents of a cask that measures 6 ft. in length, 36 inches end diameter, bung diameter 42 in.?

CLASS 4.

§ Place the dimensions as before, with 16 on the left, and 96 on the right.

EXAMPLES.

1. How many gallons will a cask hold of class 4th, of the following dimensions: diameter of head 15 inches, bung 17, length 3 feet? Ans. 32 gal.

2. How many gallons will a cask contain that is 6 feet long, 46 inches end diameter, bung 50? Ans. 576 gal.

CLASS 5.

§ Place the dimensions as before, with 16 on the left and 104 on the right.

EXAMPLES.

1. How many gallons in a cask of the 5th class, of the following dimensions: end diameter 30 inches, bung 34, length 3 feet? Ans. 138⅜ gal.

3 12	
12	64 4
2	64
2	3
10	104

$138\frac{2}{3}$

2. How many gallons in a cask of the following dimensions: end diameter 15 inches, bung 17 inches, length 3 feet?

Ans. $34\frac{2}{3}$ gal.

CLASS 6.

§ Place the dimensions as before, with 16 on the left, and 105 on the right.

EXAMPLES.

1. How many gallons will a cask of class 6 hold, that measures 3 feet in length, end diameter 15 inches, bung 17?

Ans. 35 gal.

2. How many gallons will a cask of the following dimensions hold: length 6 feet, end diameter 22 inches, bung 26?

Ans. $157\frac{1}{2}$ gal.

GRAIN MEASURE.

To find the number of bushels contained in a box or any other vessel.

§ Find the number of cubic feet that it contains, then if it is struck measure, place the dimensions of the box, cask, &c., on the right, together with 45, and place on the left 56.

EXAMPLES.

1. How many bushels of wheat will a bin contain, that measures 8 feet long, 4 feet wide, and 4 feet high?

	4
	4
	∅
7 ∅∅	45

7 | 720=102⅔ bushels.

2. How many bushels will a box hold, that measures 10½ feet long, 3⅓ feet wide, 2 feet high, struck measure?
Ans. 56¼ bu.

3. How many bushels will a store room hold that measures 45 feet long, 21 feet wide, and 16 feet high?
Ans. 12150 bu.

4. How many bushels of wheat will a storehouse hold, that measures 60 feet in length, 42 feet in width, and 20 feet high? Ans. 40500 bu.

5. How many bushels of wheat will a box contain, that measures 12 feet long, 3 feet wide, and 3½ feet deep?
Ans. 101¼ bu.

6. What is the value of 3000 pounds of wheat, at 75cts. per bushel? Ans. 37½ dollars.

2 ∅∅	300∅
4	$

37½

7. What is the value of 8425 pounds, at 60 cts. per bushel? Ans. $84,25.

8. What is the value of 2460 pounds, at 62½ cts. per pound? Ans. $25⅝.

9. What is the value of 8400 pounds of rye, at 56 cts. per bushel? Ans. $84.

10. What is the value of 22400 pounds of corn, at 28 cts. per bushel? Ans. $112.

11. What is the value of 1600 pounds of oats, at 25 cts. per bushel? Ans. $12,50

$2	1∅00
4	1

12,50

12. What is the value of 2000 pounds of oats, at 24 cts. a bushel?

THE

PRUSSIAN CALCULATOR.

PART III.

PHILOSOPHICAL ARITHMETIC.

MECHANICAL POWERS.

The MECHANICAL POWERS are commonly reckoned as six, viz: the LEVER, WHEEL and AXLE, PULLEY, INCLINED PLANE, SCREW, and WEDGE.

LEVER.

§ The Lever is an inflexible rod; such as a handspike or crow-bar; and is divided into different orders, according to the relative positions of the power, the prop or fulcrum, and the weight to be raised.

CASE 1.

To find the weight that may be raised, the power applied and length of the arms being given.

§ Place on the right of the line, the power applied and the length of the longest arm, and on the left, place the length of the short arm.

EXAMPLES.

1. What weight may be raised with a lever 12 feet long, the power to be exerted being equal to 100 lbs., and the fulcrum 2 feet from the weight? Ans. 500 lbs.

2 | 1000
  | 100

500

CASE 2.

To find the power, the weight and length of the arms being given.

§ Place the weight and length of the shorter arm, on the right; and place on the left the length of the long arm.

EXAMPLES.

1. What power must be applied on the end of a lever, 32 feet in length, to raise 1200 lbs.; the fulcrum being 2 feet from the weight? Ans. 80 lbs.

CASE 3.

To find the length of the long arm, the weight, power and short arm being given.

§ Place the weight and length of the short arm, on the right; and the power on the left.

EXAMPLES.

1. A lever is 30 feet long, and the fulcrum 10 feet from one end; what power applied at the end of the short arm will balance 120 lbs. at the end of the long arm?

Ans. 240 lbs.

2. What must be the length of the long arm of a lever, to raise a weight of 4800 lbs.; the power applied being 240 lbs., and fulcrum 6 inches from the weight?

Ans. 10 ft.

CASE 4.

To find the length of the short arm, the length of the long arm, power and weight being given.

§ Place the length of the long arm, and power, on the right; and weight on the left.

EXAMPLES.

1. What is the length of the short arm of a lever, the long arm being 24 feet, the weight 1440 lbs., and power 600 lbs. Ans. 10 ft.

2. What is the length of the short arm of a lever, the long arm being 12 feet, weight 1600 lbs. and power 400 lbs. Ans. 3 ft.

3. If a man weighing 180 lbs., rest on the arm of a lever which is 20 feet long, what power will he balance on the short arm, it being 5 feet. Ans. 720 lbs.

4. At what distance from the weight of 1440 lbs., must the fulcrum be placed so as to balance 360 lbs. on the long arm, 5 feet from the fulcrum? Ans. 20 ft.

5. Two horses are in strength as 2 to 3; how must their double-tree of $3\frac{1}{2}$ feet in length, be divided so that they may draw accordingly?

Ans. the fulcrum is $1\frac{1}{2}$ feet from the end to which the strongest horse is attached.

6. Two men carry a load on a pole of 240 lbs.; the pole is 10 feet long, and A. is six feet from the weight; what is the weight that B. sustains, being only 4 feet from the weight? Ans. 144 lbs.

7. How many lbs. will a power of 9 lbs. placed 16 inches from the fulcrum of a lever, support at the extremity of the other arm, 2 inches from the fulcrum?

Ans. 72 lbs.

8. A weight of 20 tons is suspended at the end of a lever 6 inches from the fulcrum; what power applied at the other extremity will balance, the distance being 20 feet?

WHEEL AND AXLE.

CASE 1.

The diameter of the wheel, diameter of the axle, and power being given, to find the weight.

§ Place the diameter of the wheel and power applied, on the right; and the diameter of the axle on the left.

EXAMPLES.

1. If the diameter of a wheel is 3 feet, and that of the axle 6 inches; what weight applied to the axle will balance 32 lbs. attached to the wheel? Ans. 192 lbs.

2. If the diameter of a wheel is 18 feet and that of the axle 4 inches, what weight attached to the axle, will balance 12 cwt. applied to the wheel? Ans. 648 cwt.

CASE 2.

The diameter of the wheel, diameter of the axle, and the weight being given, to find the power.

§ Place the diameter of the axle, and the weight, on the right, and the diameter of the wheel on the left.*

EXAMPLES.

1. The diameter of a wheel is 20 feet, and that of the axle 5 inches; what power applied to the wheel will balance 100 tons attached to the axle? Ans. $2\frac{1}{2}$ tons.

2. If the diameter of a wheel is 40 feet, and that of the axle is 8 inches, what power applied to the wheel will balance 140 cwt. attached to the axle? Ans. $2\frac{1}{2}$ tons.

3. If the diameter of a wheel is 16 feet, and the diameter of the axle 4 inches, what power applied to the wheel will balance 48 cwt. attached to the axle? Ans. 1 cwt.

*The other two cases may be performed without further illustration.

4. If the diameter of the axle is 4 inches, that of the wheel 28 feet, what power applied to the wheel will balance 140 tons attached to the axle? Ans. 1½ tons.

5. A wheel is 40 feet diameter, that of the axle 5 in.; what power applied to the wheel will support 12 tons attached to the axle? Ans. ⅛ ton.

INCLINED PLANE.

To find the weight, power, length or height of the Plane.

CASE 1.

The height, length of the plain and power being given, to find the weight.

§ Place the power and length of the plane on the right, and the perpendicular height on the left.

EXAMPLES.

1. What power is requisite to raise a weight of 400 pounds, up an inclined plane that is 8 feet long and 4 feet high? Ans. 200 lbs.

2. If the length of an inclined plane is 32 feet, and the height 4, what power must be applied to raise 800 pounds? Ans. 100 lbs.

3. What weight will sustain 8 tons on an inclined plane, 48 feet long and 6 feet high? Ans. 1 ton.

4. What power will balance 144 tons on an inclined plane, the length of which is 32 feet and height 4 feet? Ans. 18 tons.

7. Required the power that will balance 24 tons on an inclined plane, the length of which is 480 feet, and height 25? Ans. 1¼ tons.

CASE. 2.

The length, height of the plane and weight being given, to find the power.

§ Place the weight and height of the plane on the right, and the length of the plane on the left.*

EXAMPLES.

1. If the length of an inclined plane is 96 feet and height 12, what power will balance 144 tons on the plane?

Ans. 18 tons.

2. If the length of an inclined plane is 80 feet and the perpendicular 12, what weight must be applied to sustain 160 cwt. on the plane? Ans. 24 cwt.

3. What power will balance 12 tons on an inclined plane, 60 feet in length and height 5 feet.

Ans. 1 ton.

4. What power will balance 144 tons, on an inclined plane that is 80 feet long and 15 feet high?

Ans. 27 tons.

5. A power of 68 lbs. at the rate of 200 feet per minute, is applied to raise a weight up an inclined plane at the rate of 50 feet per minute, when the plane is 37 feet long and 12 feet high; what is the weight that can be raised?

Ans. 838$\frac{4}{9}$ lbs.

PULLEY.

There are two kinds of pulleys, the fixed and the moveable. From the fixed, when applied separately, there is no power gained.

CASE 1.

To find the weight that a given power will raise.

* The other two cases may be performed without any further illustration.

§ Place the power and the number of cords on the right.

EXAMPLES.

1. What weight will a power of 360 lbs. balance when applied to two blocks, the one of three and the other of two pullies, or when there are five cords?

Ans. 1800 lbs.

2. What weight will a power of 500 tons balance, when applied to two blocks of three pullies each?

Ans. 3000 tons.

CASE 2.

To find the power that will raise a given weight.

§ Place the number of cords that support the weight on the left, and on the right place the weight to be raised.

EXAMPLES.

1. What power must be applied to raise 180 lbs., when we have two blocks of three pullies each, or when there are 6 cords? Ans. 30 lbs.

$$6 \mid 180 \quad 30$$

2. What power will balance a weight of 9 tons, when applied to two blocks of 4 pullies each? Ans. 45 cwt.

NOTE.—Some deduction must be made for friction.

SCREW.

The screw is a cord wound round the periphery of a cylinder, and is therefore an inclined plane; the length of which is the circumference of the cylinder, and the height is the distance between the threads of the screw.

CASE 1.

The distance between the threads of the screw, the weight, and circumference of the screw being given, to find the power.

§ Place the weight and the distance between the threads of the screw on the right, and the circumference of the screw on the left.

EXAMPLES.

1. What power is requisite to raise a weight of 1200 lbs., by a screw of 12 inches circumference and 1 inch pitch? Ans. 100 lbs.

$$12 \mid 1200$$
$$1 \mid$$
$$100$$

2. What power will raise a weight of 4 tons by a screw of 24 inches circumference, and $\frac{1}{2}$ inch between the threads? Ans. $1\frac{3}{4}$ cwt.

CASE 2.

The distance between the threads of the screw, the length of the lever and power applied being given, to find the weight.

§ Place the circumference described by one revolution of the lever, and the power applied, on the right; and place on the left the distance between the threads of the screw.

EXAMPLES.

1. If the threads of a screw are $1\frac{1}{4}$ inches apart, the lever 5 feet, and the power applied at the end of the lever being 180 lbs., what weight will produce an equilibrium?

2. If the threads of a screw are 2 inches apart, and 180 lbs. be applied at the end of a lever 8 feet in length, what power will produce an equilibrium?

3. Suppose the threads of a screw be 2½ inches asunder, the lever 6 feet in length, and a power of 12 tons be applied at the end of the lever, what weight will be required to produce an equilibrium?

4. Suppose the threads of a screw to be 3 inches apart, and the length of the lever to be 20 feet, and the power applied at the end of the lever to be 1200 lbs., what weight will produce an equilibrium.

NOTE.—To get the circumference, multiply the diameter by 3,1416.

MACHINERY.

To ascertain the number of revolutions that a drum, pully, spindle, mill-stone, &c., will make when the wheels are connected together by belts, bands, cogs, &c., the velocity of the driver and diameters of the wheels being given.

§ Place the velocity and the diameter of the drivers on the right, and the diameters of the driven on the left.

EXAMPLES.

1. A cog wheel of 150 inches diameter, geers into one of 15 inches diameter; how many revolutions will the smaller make per minute, the larger performing 6?

Ans. 60.

15 | 150
6

60

2. A wheel of 40 inches diameter is connected with one of 8 inches, and this is attached to a shaft, on the end of which a wheel of 36 inches geers into one of 6 inches; what is the velocity of the last wheel per minute, the larger making 20 revolutions per minute?

3. How many revolutions will a spindle of two inches diameter make, being connected to a drum of three feet diameter, making 60 revolutions per minute?

Ans. 1080 per minute.

5. A belt connects a drum of 4 feet diameter, making 40 revolutions per minute, with one of 4 inches diameter; required the velocity of the smaller drum?

Ans. 480 per minute.

ATMOSPHERIC PRESSURE.

To ascertain the atmospheric pressure upon a cylinder or piston.

§ Place the square of the number of inches in the diameter, and 165 on the right, and 14 on the left.

EXAMPLES.

1. What is the atmospheric pressure upon a cylinder of 21 inches diameter? Ans. 5197½ lbs.

21	3
	21
2 14	165

5197½

2. What is the atmospheric pressure upon a cylinder of 56 inches diameter?

3. What is the pressure upon one of 42 inches diameter?

4. Upon one of 28? of 35? of 40? of 63?

HYDROSTATICS.

To find the pressure of water upon the bottom of a vessel.

§ Place the number of square feet contained in the bottom of the vessel, and the depth of the water and 1000 on the right, and 16 on the left.

EXAMPLES.

1. A vessel 3 feet square and 8 feet deep is filled with water; what pressure does the bottom sustain?

Ans. 4500 lbs.

	3
	3
2 1¢	¢
	1000
	4500

2. A vessel 4 feet square is filled with water to the depth of 8 feet, what pressure does the bottom sustain?

3. The area of a circular vessel is 18 feet, and the depth of water is 8 feet; what is the pressure upon the bottom?

WATER.

To find the pressure of water against the sides of gates, banks, &c.

§ Place the length and breadth of the surface against which the water presses, half the depth, and 1000, on the right; and 16 on the left.

The sides of any vessel sustain a pressure equal to the area of the sides multiplied by one-half the depth of water.

EXAMPLES.

1. What is the pressure of water against the gates of a lock when filled, that is 12 feet deep and 16 feet wide?

Ans. 72000 lbs.

	12
1¢	1¢
	6
	1000
	72000

2. What is the pressure of water on the sides of a vessel, the depth being 30 feet?

3. What is the pressure on the sides of a vessel, it being 16 feet deep and filled with water?

4. The gate of a sluice is 12 feet deep and 12 broad; what is the pressure of water against it?

5. What is the pressure of water on the sides of an aqueduct, the depth of water being 8 feet and length 120?

NOTE.—It makes no difference what the shape of the vessel is, the height of the water is all that is considered, and not its quantity.

GRAVITY.

To find the specific gravity of a body.

§ If the body is heavier than water, weigh the body in water and out of it, and the difference will be the weight lost in water.

§ Then place the whole weight and the specific gravity of water, on the right, and the loss of weight on the left.

EXAMPLES.

1. A piece of gold weighs $38\frac{9}{17}$ dwt. in the air, and $36\frac{4}{17}$ dwt. in water. What is the specific gravity?

Ans. 17 sp. gr.

17	272		
16	17	$38\frac{9}{17}$	
	1	$36\frac{4}{17}$	
	17		$2\frac{5}{17}$

2. What is the specific gravity of a body that weighs $16\frac{3}{8}$ lbs. in water, and $20\frac{1}{8}$ in air? Ans. $4\frac{1}{7}$ sp. gr.

*The specific gravity of water is 1. A cubic foot of water weighs 1000 ounces avoirdupois.

DISTANCE.

To determine the distance of a gun or a thunder cloud, from seeing the flash and hearing the report.

§ Place the number of seconds that elapse between the flash and the report, and 1142* on the right, and place the next inferior numbers that correspond with the answer, on the left; or if you count the pulsations, place them on the right, together with 1142 and 60; and place on the left 72, (the number of pulsations in one second,) and the next inferior denominations that correspond with the answer.

EXAMPLES.

1. After seeing the flash of a cannon, 30 seconds elapse; what was the distance? Ans. 6,53 miles.

16 ~~320~~	~~30~~
	1142
11 ~~33~~	~~2~~
6,53	

2. What is the distance of an electrical cloud, the flash being seen 24 pulsations before the report is heard?

Ans. 4,33 miles nearly.

	~~24~~
~~72~~	~~60~~
8 ~~320~~	1142
,33	~~2~~
4,33	

3. I observed a meteor burst, and 160 seconds after heard the report; what was the distance of the meteor from the place where I stood?

Ans. 34,6+miles.

* The number of feet that sound travels in a second of time.

GRAVITATION.

To find how far a body falls in a given time, if it meets with no resistance.

§ Place the square of the time in seconds, and 193,* on the right; and 12 on the left.

EXAMPLES.

1. A body has been descending 6 seconds; how far has it fallen in that time? Ans. 579 ft.

	6
12	6 3
	193
	579

2. From a dark cloud a flash of lightning was observed, and 12 seconds after, the rain struck the ground; what was the height of the cloud, if the rain commenced descending when the flash was observed? Ans. 2316 ft.

3. What is the distance a body will fall in 16 seconds? Ans. 4117½ ft.

	193
3 12	16
	16 4
	4117½

4. How far will a body fall in 32 seconds?
Ans. 16469½ ft.

* The distance a body falls in a second of time, is 16 and 1-12th feet.

THE

PRUSSIAN CALCULATOR.

APPENDIX.

REMARKS.

§ It is presumed that you have become acquainted with the method of solving problems, as presented in the pages of this work. This being the case, the author will now present many interesting questions, and their solutions, thus leading you through paths strown with fragrant flowers; and will give you a still more extensive knowledge of the powers of numbers.

Let not the difficulties that may present themselves in your path, debar you from entering the Elisian bower, or from drinking deep at the fountain of knowledge.

He is unworthy the name of man, who having the opportunities of gaining knowledge, neglects to improve them.

Let those who have passed the spring-time of life, not forget that they may, by improving their spare moments, yet regain in a great measure, their lost opportunities. A very short period of each day, devoted to the study of this useful science, will give you a practical knowledge of Business Arithmetic.

§ In commencing my remarks in the Appendix, on the different methods of solving questions, I shall in the first place present you with a series of numbers which are called *Primes;* so called because no number greater than 1 can measure or divide them.

But you will observe that these numbers may often be used as divisors of other numbers.

Primes.—1.2.3.,.5.,.7.,.,.,.,.11.,,.13.,,.,.,.,.,.17.,,. 19.,,.,.,.,.,23.,,.,.,.,.,.,.,.,.,.29.,,.31.,,.,.,.,.,.,.,.,.37. ,,.,.,.,.,.41.,,.43.,,.,.,.,.,.47.,,.,.,.,.,.,.,.,.,.53.,,.,.,. ,,.57.,,.59.,,.61.,,.,.,.,.,.,.,.,.,.67.,,.,.,.,.,.71.,,.73. ,,.,.,.,.,.,.,.,.,.79.,,.,.,.,.,.,.83.,,.,.,.,.,.,.87.,,.89.,,. ,,.,.,.,.,.,.,.,.,.97.,,.,.,.,.,.,.,.

You will observe that we have 28 *primes* in the first 100, and as we ascend, the *prime* numbers become less. In the first 100 we have 72 composite numbers. When primes occur, we cannot reduce by canceling, without having fractions; and whenever we have primes, both for divisor and dividend, the answer will involve a fraction.

You will observe the commas in the above series between the primes. These stand for the composite numbers.

Between one and twenty-five we have 9 primes, and counting the one, we have 10.

Between 25 and 50 we have 6; and between 50 and 75 there are 7; and between 75 and 100 there are 5. We find, then, that the primes in the first 100 are to the composite numbers as about one to four; yet in business these will not occur as frequently as in the series.

In business, the principles of canceling will apply in 9 cases at least out of ten.

§ It would be well for those who examine and practice this rule, to become familiar with the following

Scale—1, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, &c.

§ In this scale you will observe that each number is some aliquot part of the other. For illustration, 5 is one-half of 10; 15 is one-fourth of 60; and 1 is one-fifth of 5, &c., &c.

§ You may now be ready to inquire, what is the object of the above tables. It is this: that the eye may become familiar with the series, so that we may readily distinguish between the prime and composite numbers.

§ You have already become acquainted with contractions to some extent. We shall now present you with many other contractions in solving questions. The ready application of contractions, marks the distinction between the reasoning and automatic mathematician.

He that uses contractions shows a scientific knowledge of numbers, while he who follows a dead uniformity of operations, shows the mechanical.

§ The first thing that we shall present, is

ADDITION.

In Addition, very little contraction can be made.

It is true that our system of Addition is vastly superior to that used by the ancients, and perhaps the present method of increase by tens, is the best that has been or ever will be used.

§ It has been proposed by some to use the sexagesimal or increase by 60, in place of 10.

If this mode was adopted, we should have to use 60 different characters, in place of 10.

When we wish to add by the sexagesimal, we add one for every sixty to the next left hand column. We have an example of this in the measure of time, as 60 seconds make a minute, 60 minutes an hour, &c.; so with the circle: but as 60 is also a multiple of 10, it embraces both the decimal and sexagesimal in the operation. We have also the remnants of many other systems of Division; and those too, with little or no regularity.

For illustration, we may take the different weights, measures, coins, &c.

If these were all reduced to the decimal system, the science of Arithmetic would be very easily learned, as will be seen from the readiness with which calculations are made in federal money.

In business we have frequent occasion to use Addition, and as the accuracy of almost every problem depends more or less for correctness on the accuracy of our additions, great care should be taken to have them correct.

In Addition, some perform the operation by adding upwards and then downwards.

Thus :
164
276
898
552
527
733
3150

Commencing at the right, first adding the several numbers, commencing with the 3; say 3 and 7 are 10 and 2 are 12, 12 and 8 are 20, 20 and 6 are 26, 26 and 4 are 30. Set down the 0 and carry the 3; then add the next column as before, and thus proceed through, setting down the result. Then commence at the top, at the right, and add downwards as before, setting down the result.—If the two answers agree, it is presumed that the work is right; as it would hardly ever occur that we would make the same error in adding both ways.

Some accountants also practice adding the columns separately, and then adding the result, as in the example.

486
587
688
799
841
3401

They would find the amount in the first column, then in the second, and so on; as 1+9+8+7+6 = 31 units,
and 4+9+8+8+8 = 37 tens,
and 8+7+6+5+4 = 30 hundreds.
3401

Others again, add by calling several numbers at a time, as 6 and 8 and 4 are 18, and 18 and 6 and 4 and 8 are 36, &c.; While others add two columns at a time, as 21 and 40 are

61, and 92 are 153, and 153 and 65 are 218, &c. I have also observed some of the best accountants add by tens; or as we might with more propriety say, they add till the amount of their figures equals 10, then they drop 10 and continue to add till the amount again equals 10; they then drop and proceed as before, making a point with the pen or pencil, at each figure where the sum equals 10 :—thus, in the example, 7 and 3=10·, 2 and 8=10·, 6 and 4=10·, place the cipher below, count the tens, and carry them to the next column; then add as before, saying : 3 and 2+5+3= 13; drop 10 and carry 3; 3+9=12; drop 10 and carry 2; 2+7+6=15 ; make a point at 5, at 9, and 6; place the 5 under the column; then count the tens which are 3, and place the 3 at the left of the 5, &c.

6·4·
7 6
9·8·
5·2
3 3·
2 7

3 5 0 Ans.

MULTIPLICATION.

EXAMPLES.

§ In multiplying, it is easier to multiply by 2, 3, 4, and 5, than by 7, 8, or 9, &c.

We shall now present examples in Multiplication.

1. Multiply 428 by 15.

428×15
2140

6420

§ We place the 15 at the right of 428, and use the sign of Multiplication; but this is not necessary, from the fact that it may be placed any where or not written at all ; this of course is left to the choice of the operator.

We first multiply by 5, placing the first product figure one place to the right; 5 times 8 is 40 ; then 5 times 2

equal 10, and the 4 that we carried=14, write the 4 under the 8; thus proceed; then add the two products for the answer.

2. Multiply 8844 by 14.

8844×14
35376

123816

3. Multiply 6872×16.

6872×16
41232

109952

§ The reason of the above examples is plain. It is this: the first figures of the multiplier and multiplicand are both units, and units multiplied by units, produce units; and units multiplied by tens produce tens, &c.; 15 in the first example=10+5. Multiplying by 5 increases the multiplicand 5 times, and multiplying by 10 increases it ten times; or it removes the figures one place to the left, which is the same thing.

4. Multiply 4892 by 41.

4892×41
19568

200572 Ans.

§ As multiplying by one will not increase the multiplicand, we multiply by the 4 tens, or 40. For the answer we add this product to the multiplicand.

5. Multiply 4532 by 148.

4532×148
18128
36256

670736 Ans.

§ Commence with the 4; let the multiplicand stand, and place the first product figure one place to the right. Then multiply the product of four times 4532, which is 18128, by 2; placing the first product figure one place farther to the right. Add these numbers for the answer.

Observe that 148=100+40+8; then 100 times 4532=453200, and 40 times 4532=181280, and 8 times 4532=

86256; then add these products and we have the answer.

453200
181280
36256

670736 Ans.

6. Multiply 48221 by 421.

48221×421
96442
192884

20301041 Ans.

§ As multiplying by 1 will not increase the multiplicand, we let it stand as the product by 1; then we multiply this product by 2, from the fact that 2 is twice 1; observing to place the first product figure in ten's place. Then multiply the last product by 2, placing the first product figure in hundred's place.

7. Multiply 7828 by 412.

7828×412
15656
31312

3225136

412=400+10+2

EXPLANATION.—412=400+10+2. We first multiply by 10, or which is the same thing, we let 8 stand in ten's place; for, annexing the cipher to 7828, only serves to remove the figures one place to the left; we then multiply the 7828 by 2, placing the first figure one remove to the right. Then multiply by 400, or which is the same thing, multiply the last product, 15656, by 2; observing to place the first figure in hundred's place, as 4, the multiplier, stands in hundredth's place.

8. Multiply 87284 by 124.

87284×124
174568
349136

10823216 Ans.

9. Multiply 64827 by 36.

64827×36

194481
388962

2333772

§ Commence with 3, then multiply that product by 2, placing the first product figure in the place of units.

10. Multiply 87234 by 39.

87234×39

261702
785106

3402126

11. Multiply 1876 by 186.

1876×6 First by 6, then by 3 tens or 30.

11256×30
33768

348936

12. Multiply 68944 by 104.

68944×100+4
275776

7170176 Ans.

13. Multiply 9487 by 618.

9487×600+18

56922×3
170766

5862966 Ans.

§ Multiply first by 600, then that product by 3, placing the first product figure two places to the right.

14. Multiply 728 by 399.

$728\times300+90+9$

2184×30
6552×1
6552

290472 Ans.

15. Multiply 60877 by 497.

$60877\times400+90+7$

426139
2982973

30255869 Ans.

§ Observe that $49=7\times7$. First multiply by 7, then that product by 7, placing the first figure in ten's place.

16. Multiply 428 by 729.

$428\times720+9$

3852×80
30816

312012 Ans.

17. Multiply 729 by 428.

729×428

2916×7^{0}
20412

312012 Ans.

§ Multiply first by 4, and then that product by 7.

18. Multiply 6872 by 144012.

6872×144012

82464
989568

989650464

§ Multiply first by 12, and that product by 12; observing to place the quotient figures in the right place.

19. Multiply 728 by 366.

728×366

4368
26208

266448 Ans.

20. Multiply 8421 by 728.

8421×728

58947
235788

6130488 Ans.

21. Multiply 14 by 14.

14
14

196

§ Place the multiplier one place to the left. Then square the first product figure; then take twice the product of 4 times 1, then square the last product figure.

22. Multiply 45 by 45.

45
45

2025

§ Say 5 times 5 is 25; write the 5 and carry the 2; then say 5 times 4 is 20, and twice 20 is 40, and 2 are 42; then write 2 and reserve the 4 for the next place; then, 4 times 4=16, and 16+4=20.

23. Multiply 87 by 87.

87
87

7569 Ans.

24. Multiply 92 by 92.

92
92

8464 Ans.

25. Square 82.

82
82

6724 Ans.

§ First square the two, then multiply the $2\times8=16$ and $2\times16=32$; write the two, then square the 8 and add the three.

26. Multiply 66 by 66.

66
66

4356 Ans.

27. Multiply 88 by 88.

88
88

$968\times8=7744$ Ans.

28. Multiplv 2872 by 25.

4 | 287200
25

71800 Ans.

§ Add two ciphers to the multiplicand, and divide by 4.

29. Multiply 65500 by 50.

2 | 655000

327500 Ans.

§ Add a cipher and divide by 2.

30. Multiply 54428 by 99.

$54428\times100-1=99\times54428$. § As 99 is but one less than 100, we multiply by 100; that is, we add two ciphers

to the multiplicand, and then subtract once the multiplicand.

5442800
—54428

5398372 Ans.

31. Multiply 99 by 99.

99=100—1
99=100—1

891 10000
891 —200
— +1
9801 —
9801

§ As 99 is equal to 100, less 1, we multiply 100 by itself; then we subtract twice 100 from the product, and add one. It is true that this process is no shorter than the usual method, yet in many examples it is easier. You will observe the signs that stand before these numbers; (+) this signifies to add; (—) this to subtract.

32. Multiply 89 by 89.

First method: 89
89
7921 Ans.

Second:
89=90—1
89=90—1

8100
—180
+1

7921 Ans.

Third: 89
89

801
712

7921 Ans.

33. Multiply 19 by 19.

First method: 19
19

171
19

361 Ans.

Second: 19
19

361

Third:
19=20—1
19=20—1

400
—40
+1

361 Ans.

34. Multiply 68 by 68.

First : 68
68
544
408
4624 Ans.

Second :
68
68
4624 Ans.

Third :
68=70—2
68=70—2
4624 Ans.

35. Multiply 872 by 98.

872×100—2
87200
—1744
85456 Ans.

36. Multiply 53 by 53.

First: 53
53
159
265
2809 Ans.

Second : 53
53
2809 Ans.

Third : 53=50+3
53=50+3
2500
+300
+9
2809 Ans.

37. Multiply 77 by 77.

First : 77
77
539
539
5929 Ans.

Second : 77
77
5929 Ans.

Third :
77
77
847×7
5929 Ans.

Fourth :
80—3=77
80—3=77
6400
—480
+9
Ans. 5929

§ From the above examples you can have a choice.

38. Multiply 997 by 997.

997=1000—3
997=1000—3

1000000
—6000
+9

994009 Ans.

39. Multiply 97345 by 9875.

80 | 973450000
−12168125
961281875 Ans.

§ 9875=10000—125. 125 is $\frac{1}{80}$ of 10000; hence we subtract $\frac{1}{80}$ from 973450000 for the answer.

40. Multiply 78544 by 25.

$25=\frac{4}{100}$

4 | 7854400

1963800 Ans.

41. Multiply 450 by 80, and divide the product by 90.

900 | 4500
80

400 Ans.

42. Multiply 1646 by 365.

823
730

24690
5761

600790 Ans.

§ As the first factor is even and the other odd, we mentally halve the one and double the other; the product then of the numbers will be the same.

43. Multiply 787 by 279.

787×9=7083×30
21249

219573 Ans.

44. Multiply 764321 by 64432.

764321
64432

3057284
24458272
48916544

49246730672 Ans.

§ Multiply first by 4, then that product by 8, and the last product by 2; add these products for the answer.

45. Multiply 32433621 by 9631248.

First Solution: 32433621
9631248

32433621
64867242
129734484
259468968
97300863
194601726
291902589

312376247389008 Ans.

Second: 32433621
9631248

389203452
1556813808
291902589
2043318123

Ans.

§ In the last example we first multiply by 12, and that product by 4; then we multiply by 9, and that product by 9.

46. Multiply 888 by 15, and that product by 5.

888 22200
5
15 3

66600 Ans.

222 30×10=300
300

66600 Ans.

§ Mentally take $\frac{1}{4}$ of 888; then twice 15=30, and 2×5=10

4 | 888 222
15×2=30
5×2=10 and 10×30=300

47. Multiply 12 by 25×15×5×24.

2 | 12=6
25×2=50
15×2=30
5×2=10
4 | 24=6

6×50×30×10×6=540000 Ans.

48. Multiply 65 by 65.

65
65
4225 Ans.

§ Square the 5; then take twice the product of 6×5; then square the last term. The square of the first, twice the product of the middle, the square of the last term is the process.

49. Multiply 86 by 86.

86
86
7396 Ans.

50. Multiply 444 by 444.

+444
+444
+444
49284×4
197136

or,

12321
444
414
197136 Ans.

§ You will observe the figures placed over the line in the last example. First multiply 4 by itself; then twice the product of the next by itself. By following the numbers ov-

or the line, you will find no difficulty in squaring figures of like denominations by one operation.

51. Multiply 22222 by 22222.

$$\begin{array}{rl} & {\scriptstyle 1\,2\,3\,4\,5\,4\,3\,2\,1} \\ & 22222 \\ \textit{Solution:} & 22222 \\ \hline & 493817284 \text{ Ans.} \end{array}$$

52. Multiply 960 by 720.

Divide both by 120 and then we have 6 and 8; and 6 times 8=48×12×12×100=691200. Ans.

53. Multiply 440 by 880. 387200 Ans.

TO MULTIPLY IN ONE LINE.

54. Multiply 2334421 by 1423322.

$$\begin{array}{rrrrrrr} 2^{7} & 3^{6} & 3^{5} & 4^{4} & 4^{3} & 2^{2} & 1^{1} \\ 1^{6} & 4^{5} & 2^{4} & 3^{3} & 3^{1} & 2^{1} & 2^{0} \end{array}$$

$$3^{13}3^{12}2^{11}2^{10}6^{9}3^{8}2^{7}7^{6}3^{5}6^{5}5^{3}3^{2}2^{1} \text{ Ans.}$$

§ According to this rule we say, two times 1 is 2: set down the 2. Then say 2×2 is 4, and 2 times 1 is 2, and 4 and 2=6: set down the 6 in the ten's place; then say, 3 times 1=3, 2×2=4, and 2×4=8, and 3+4+8=15. Then set down the 5 in the third place and carry the 1. Then say, 3×1=3, and 3×2=3, and 2 times 4=8, and 2×4=8, and 1+3+6+8+8=26: set 6 in the fourth place, and carry the 2, &c. Observe the index figures that are placed at the right of the multiplicand and multiplier. The first index figure in the multiplicand is 1, and in the multiplier it is a cipher (°). The addition of these makes 1; the product of the first figures therefore is put in the unit's place. Hence you will observe that the addition of these index figures, indicates where the product of any two figures is placed.

§ The *multiplication* by this rule is simple and useful, as it saves time and paper; and it is valuable because of the facility it gives in *addition*, and is a beautiful exercise for the memory.

55. Multiply 432132 by 432132.

$4^6\ 3^5\ 2^4\ 1^3\ 3^2\ 2^1$
$4^5\ 3^4\ 2^3\ 1^2\ 3^1\ 2^0$

$18^{11}6^{10}7^{9}3^{8}8^{7}0^{6}6^{5}5^{4}4^{3}2^{2}4^{1}$ Ans.

56. Multiply 5642 by 369.

5642
369

16926
33852
50778

2081898 Ans.

§ In this example we commence multiplying with the 3. First, multiply 56 by 3, and the product is 168; but the product of 4 by 3 gives us one to carry to 168, and we have 169, which we write, placing 1 beneath the 5: we then see by a glance of the eye, that 26 are the next two right hand figures. Then multiply 16926 by 2, saying 2 times 16=32, and by inspection we find there will be one to carry=to 33; write 33, placing the first left hand figure one place to the right; then say, 2 times 92 =184, and 1 to carry=185; and at the same time we see that 2 will be the first right hand figure. The last product is obtained by multiplying the first, in the same way, by three.

57. Multiply 2560 by 393.

2560
393

7680
23040
7680

1006080 Ans.

§ Multiply 25 by 3=75, and as we see at a glance that the next figures will be 18, we write at once 7680; then proceed as in the last example.

58. Multiply 148621 by 245682.

$1^6\ 4^5\ 8^4\ 6^3\ 2^2\ 1^1$
$2^5\ 4^4\ 5^3\ 6^2\ 8^1\ 2^0$

$3^{11}6^{10}5^9 1^8 3^7 5^6 0^5 4^4 5^3 2^2 2^1$ Ans.

59. Multiply 65 by 65.

65
65
4225

§ To square 65: first square the 5, which gives 25; then double the 6=12; then multiply the 12 by 5=6; then square the 6=36; and then add 6, the left hand figure of the product of 12 by 5.

§ We can also square numbers by squaring the first figure, placing the unit figure in unit's place; then double the next figure in the multiplicand, and multiply it by the first figure of the multiplier. Then add the tens that were produced by multiplying the units; then call the ten in the multiplier one greater than it is. For illustration:

75
75
5625

§ First square the 5=25; write this for units and tens; then call the 7 tens in the multiplicand 8, and multiply this by 7=56; write this for hundreds and thousands.

DIVISION.

REMARKS.

§ The principles of Division have been sufficiently illustrated in the foregoing pages, so far as simple numbers are concerned. Hence, no farther illustrations on that point are necessary; but we will make a few remarks on Division of Decimal Fractions.

§ Fractions, you have learned, are parts of a unit; or, in other words, they bear some proportion to another num-

ber which is taken as the unit figure. For illustration: one cent is some part of five cents; but one cent is just as much a whole number as one dollar is. Also, one inch is a part of one foot; yet it is as much a whole thing as one foot is.

§ You have observed that numbers are frequently written thus: 4,50; 6,25; 6,8; which may be read, four and fifty one-hundredths; 6 and twenty-five one-hundredths; 6 and eight-tenths. The denominator of these numbers is not written or expressed; and for this reason, the denominator is 10, or a multiple of 10, by itself, (hence, the only difference between these fractions and those that have denominators written, consists in this.)

§ The reduction of a *common* to a *decimal* fraction, consists in dividing the numerator by its denominator, which is done by placing the numerator on the right and annexing ciphers, and then dividing by the denominator, which is placed on the left; observing to place the decimal point in its proper position.

Reduce $\frac{1}{2}$ to a decimal.

2 | 1,0

,5 Answer, five-tenths.

FRACTIONS.

The following examples are inserted for practice; and to those who are not familiar with fractions, we would recommend a careful examination of them.

§ Fractions are divided into two classes, called VULGAR and DECIMAL.

A vulgar fraction is a part or parts of some other number or numbers, and is expressed by two numbers placed one above the other, with a line drawn between them; as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c.

The number placed below the line is the denominator, and that above the line is the numerator.

The denominator shows into how many parts the unit is supposed to be divided.

The numerator shows how many parts are supposed to be taken or used.

Fractions are either PROPER, IMPROPER, COMPOUND, *or* COMPLEX.

A proper fraction is one whose numerator is not greater than unity, as $\frac{5}{8}$; $\frac{4}{7}$; *or* $\frac{5}{6}$; *&c.*

An improper fraction is one whose numerator is greater than unity, as $\frac{7}{3}$; $\frac{9}{4}$; $\frac{8}{3}$; *&c.*

A compound fraction is a fraction of a fraction, as $\frac{1}{2}$ *of* $\frac{2}{3}$; *&c.*

A complex fraction is one which has a fraction in either or both the numerator and denominator, as $\frac{\frac{1}{2}}{\frac{2}{4}}$ *or* $\frac{\frac{6}{7}}{1\frac{1}{2}}$.

A mixed number is a whole number joined to a fraction, as $4\frac{1}{2}$ *or* $6\frac{1}{7}$.

REDUCTION OF FRACTIONS.

CASE FIRST.

To reduce a fraction to its lowest denominations.

§ Place the numerator on the right and denominator on the left. Then divide by any number that will divide both without a remainder, and then divide the quotients till no number greater than unity will divide both without a remainder. The fraction then is in its lowest terms.

EXAMPLES.

1. Reduce $\frac{9}{36}$ to its lowest terms.

36 | 9
4 | 1

§ Divide both by 9, and we have as the result, one-fourth; from which it is evident that 9 is $\frac{1}{4}$ of 36.

2. Reduce $\frac{120}{190}$ to its lowest terms.

120 | 190	§ Divide by 10.	Ans. $\frac{12}{19}$.

3. Reduce $\frac{864}{1008}$ to its lowest terms. Ans. $\frac{7}{8}$.

4. Reduce $\frac{48}{56}$ to its lowest terms. Ans $\frac{6}{7}$.

CASE SECOND.

When a divisor cannot be readily found.

EXAMPLES.

1. Reduce $\frac{85}{136}$ to its lowest denomination. Ans. $\frac{5}{8}$.

1 136	85	1	§ First, we divide the denominator by
85	51		the numerator, and find that 85 is con-
—	—		tained in 136 once; we then subtract 85
1 51	34	2	from 136, and then divide 85 by that
34	34		remainder, and find that 51 is contained
—	—		in it but once; we now subtract 51 from
17	00		85, and then divide 51 by this remainder,
—	—		so continue till nothing remains. The
8	5		last divisor is the greatest common divi-

sor, and this in the example is 17, and with it we divide 136 and 85.

2. Reduce $\frac{91}{117}$ to its lowest terms. Ans. $\frac{7}{9}$.

1 117	91	3	
91	78		§ 13 is the common measure.
—	—		
2 26	13		
26			

4. Reduce $\frac{144}{216}$ to its lowest terms. Ans. $\frac{2}{3}$.

1 216	144	2	
144	144		§ 72, common measure or divisor.
—	—		
72	000		

4. Reduce $\frac{2841}{6231}$ to its lowest terms. Ans. $\frac{11}{19}$.

CASE THIRD.

To reduce a mixed number to an improper fraction.

§ Multiply the whole number by the denominator, and add in the numerator.

EXAMPLES.

1. Reduce $12\frac{4}{9}$ to an improper fraction. Ans. $\frac{112}{9}$

$$9 \mid 12\frac{4}{9}$$
$$112$$

2. Reduce $17\frac{3}{7}$ to an improper fraction. Ans. $\frac{122}{7}$
3. Reduce $45\frac{2}{3}$ to an improper fraction. Ans. $\frac{137}{3}$

CASE FOURTH.

To reduce an improper fraction to its true value.

EXAMPLES.

1. Reduce $\frac{17}{5}$ to proper terms. Ans. $3\frac{2}{5}$

$$5 \mid 17$$
$$3\frac{2}{5}$$

2. Reduce $\frac{140}{9}$ to a mixed number. Ans. $15\frac{5}{9}$
3. Reduce $\frac{245}{3}$ to a mixed number. Ans. $81\frac{2}{3}$
4. Reduce $\frac{43}{8}$ to proper terms. Ans. $5\frac{3}{8}$
5. Reduce $\frac{98}{12}$ to a mixed number. Ans. $8\frac{1}{6}$
6. Reduce $\frac{1003}{572}$ to a mixed number. Ans. $8\frac{1}{8}$
7. Reduce $\frac{1}{5}\frac{7}{5}$ of $\frac{45}{9}$ to a mixed number. Ans. 27

CASE FIFTH.

To reduce several fractions to other fractions having a common denominator.

EXAMPLES.

1. Reduce $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ to a common denominator.

	72
3	1=24
4	1=18
6	1=12
	$\frac{6}{7}\frac{4}{2}$

§ Multiply the denominators together, and place their product on the right; and then divide this product by each denominator, and multiply the quotient by each numerator separately, commencing at the top. Thus, in the example, the product of the denominators is 72; and $\frac{1}{3}$ of 72 is 24; and $\frac{1}{4}$ is 18; and $\frac{1}{6}$ is 12; and their sum is $\frac{6}{7}\frac{4}{2}$.

2. Reduce $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{12}$ to a common denominator.

	288
4	3=216
6	5=240
12	7=168
	$\frac{6}{2}\frac{2}{8}\frac{4}{8}$ or, $2\frac{4}{2}\frac{3}{8}\frac{3}{8}$

Ans. $\frac{6}{2}\frac{2}{8}\frac{4}{8}$ or, $\frac{2}{2}\frac{4}{8}\frac{3}{8}$

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{8}$ to a common denominator.

Ans. $\frac{1}{2}\frac{2}{4}$, $\frac{1}{2}\frac{6}{4}$, $\frac{2}{2}\frac{0}{4}$, $\frac{2}{2}\frac{1}{4}$

CASE SIXTH.

To find the least common denominator.

EXAMPLES.

1. Change $\frac{1}{6}$ and $\frac{1}{9}$ to a fraction having the least common denominator.

3)	18
2 6	1=3
3 9	1=2
	$\frac{5}{1}\frac{5}{8}$

§ Divide by 3.

2. Change $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{4}{5}$ to equivalent fractions, having the least common denominator. Ans. $\frac{41}{24}$

	24
2	4=12
3	3= 9
4	5=20
	$\frac{41}{24}$

§ First, we divide by any number that will divide two or more of the denominators without a remainder; so continue till we can divide no longer. We then multiply the quotients and divisors together, and their product is the least common denominator.

3. Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$ to equivalent fractions having the least common denominator. Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

4. Change $\frac{3}{4}$, $\frac{5}{12}$, $\frac{2}{3}$ and $\frac{1}{6}$ to equivalent fractions having the least common denominator. Ans. $\frac{9}{12}$, $\frac{6}{12}$, $\frac{8}{12}$, $\frac{2}{12}$.

CASE SEVENTH.

To change a compound fraction to an equivalent single one.

EXAMPLES.

1. Change the compound fraction $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{4}{5}$ to a single one. Ans. $\frac{1}{4}$.

2	1
4	3
2 6	5
5	4
4	1=$\frac{1}{4}$

§ Place the numerators on the right, and the denominators on the left.

2. Change $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ to an equivalent fraction. Ans $\frac{3}{7}$.

3. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{1}{8}$ of $\frac{8}{12}$ of $\frac{4}{5}$ to its lowest terms. Ans. $\frac{1}{24}$.

CASE EIGHTH.

REDUCTION OF FRACTIONS FROM ONE DENOMINATION TO ANOTHER.

To change a fraction of a higher, into a fraction of a lower denomination.

EXAMPLES.

1. Reduce $\frac{1}{240}$ of a £ to the fraction of a penny.

Ans. $\frac{5}{6}$ of a penny.

6 ~~24~~ ~~240~~ | 1
~~12~~
~~20~~ 5

| $\frac{5}{6}$ of a penny.

2. Reduce $\frac{1}{320}$ of a £ to the fraction of a penny.

Ans. $\frac{3}{4}$.

4 ~~16~~ ~~320~~ | 1
~~20~~
~~12~~ 3

4 | 3

3. Reduce $\frac{1}{3360}$ of a £ to the fraction of a farthing.

Ans. $\frac{2}{7}$.

7 ~~14~~ ~~3360~~ | 1
~~20~~
~~12~~
~~4~~ 2

| $\frac{2}{7}$

4. What part of a pound is $\frac{1}{300}$ of a cwt. Ans. $\frac{1}{3}$.

5. Reduce $\frac{1}{24}$ of a yard to the fraction of a nail.

Ans. $\frac{3}{4}$.

6. Reduce $\frac{1}{1920}$ of a £ to the fraction of a farthing.

Ans. $\frac{1}{2}$.

CASE NINTH.

To reduce fractions of a lower denomination to those of a higher.

EXAMPLES.

1. Reduce $\frac{5}{6}$ of a penny to the fraction of a £.

Ans. $\frac{1}{288}$.

6 | 5
12 |
4 20 |
288 | 1

2. Reduce $\frac{3}{4}$ of a farthing to the fraction of a £.

Ans. $\frac{1}{1920}$.

3. Reduce $\frac{4}{7}$ of a guinea to the fraction of a £.

7 | 4 Ans. $\frac{4}{5}$.
1 | 20
5 20 |

4. What fraction of a £ is $\frac{4}{5}$ of a guinea? Ans. $\frac{4}{7}$.
5. Reduce $\frac{5}{1080}$ of a £ to the fraction of a farthing.

Ans. $\frac{4}{9}$.

6. What part of a quart is $\frac{1}{24}$ of a bushel. Ans $\frac{2}{3}$.
7. Reduce $\frac{1}{20}$ of a yard to the fraction of a nail. Ans. $\frac{4}{5}$.
8. Reduce $\frac{7}{1280}$ of an acre to the fraction of a perch.

Ans. $\frac{7}{8}$.

9. Reduce $\frac{1}{72}$ of a hogshead to the fraction of a pint.

Ans. $\frac{8}{9}$.

10. Reduce 3 qrs. 2 na. to the fraction of a yard.

Ans. $\frac{7}{8}$.

4 | $\overbrace{4\times3+2}$: or, 4 | $\overbrace{3\times4+2}$.
4 | 4 |
8 | 7 $\frac{14}{16}=\frac{7}{8}$

CASE TENTH.

To add fractions.

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ together. Ans. $1\frac{11}{12}$.

	12
2	1=6
3	2=8
4	3=9

$\frac{23}{12}$

2. Add $\frac{3}{5}$, $\frac{4}{6}$ and $\frac{5}{8}$ together. Ans. $2\frac{1}{120}$.

CASE ELEVENTH.

To subtract vulgar fractions.

EXAMPLES.

1. Subtract $\frac{3}{12}$ from $\frac{3}{4}$. Ans. $\frac{1}{12}$.

	12
4	3=9
12	3=8

$\frac{1}{12}$.

2. Subtract $\frac{4}{6}$ from $\frac{3}{4}$. Ans. $\frac{1}{12}$
3. Subtract $\frac{7}{12}$ from $\frac{4}{6}$. Ans. $\frac{1}{4}$
4. Subtract $\frac{2}{5}$ of $\frac{7}{12}$ from $\frac{13}{20}$. Ans. $\frac{1}{5}$

CASE TWELFTH.

To subtract fractions.

EXAMPLES.

1. Subtract $\frac{4}{5}$ from $\frac{4}{5}$. Ans. $\frac{1}{5}$

4	§ Where the denominators are alike, subtract the numerator; but where they are not alike, find a common denominator and subtract the numerators.
5	
Ans. $\frac{1}{5}$	

2. Take $\frac{2}{5}$ from $\frac{3}{7}$. Ans. $\frac{1}{35}$

	35
5	2=14
7	3=15

$\frac{1}{35}$

3. From $\frac{3}{4}$ of a £ take $\frac{3}{4}$ of a shilling.

Ans. $14\frac{1}{4}$ shillings.

CASE THIRTEENTH.

To multiply fractions.

EXAMPLES.

1. Multiply $\frac{4}{5}$ by $\frac{3}{16}$. Ans. $\frac{1}{12}$

3	4
4	16
12	1

2. Multiply $1\frac{1}{2}$ by $\frac{2}{3}$. Ans. $\frac{1}{3}\frac{1}{2}$

CASE FOURTEENTH.

To divide fractions.

§ Place the dividend as in multiplying, but change the divisor; placing the numerators on the left and the denominators on the right.

EXAMPLES.

1. Divide $\frac{4}{7}$ by $\frac{2}{3}$. Ans. $\frac{6}{7}$

7	4 2
2	3

$\frac{6}{7}$

2. Divide $\frac{3}{8}$ by $\frac{3}{4}$. Ans. $1\frac{1}{6}$

DECIMALS.

To change vulgar to decimal fractions.

§ It has been seen in notation, that the local value of figures change according to the place they occupy; that every figure designates a quantity 10 times greater for every time it is removed one place to the left. The increase from right to left is tenfold. Now, it is obvious, that if we wish to designate quantities less than unity, we must place the figure or figures at the right of unit's place; and, that the notations of such numbers may correspond with whole numbers, we write them at the right hand of unit's place, (that being the lowest place of whole numbers,) and then we make them decrease in a tenfold ratio towards the right.

On this system, the first place is called tenths of units; the second place, hundredths; the third thousandths, &c.

In the system of decimal fractions, the denominator is not written, but is understood; and is always 10, or some multiple of 10 by itself; as $10\times10=100$; $10\times10\times10=1000$.

It is also desirable to have some mark of distinction between whole numbers and decimals, and this is done by a dot or comma; as 4,5: 63,35: read, four and five tenths; 63 and 35 hundreths. From this, it will be seen that decimal fractions have only the numerators written.

EXAMPLES.

1. Change $\frac{1}{2}$ to a decimal fraction.

2 | 1 § To do this, place the 1 on the right and 2 on the left: then divide by 2; but as one cannot be divided by 2, we place a cipher at the right. Thus: 1,0 with the point before it; and as we find that 2 will not divide the one, we call it 10 tenths and then divide, placing the point at the left of the ,5.

2 | 1,0
– | —
,5

To illustrate this, we will suppose that both 1 and 2 were reduced to tenths, and we have $\frac{10}{10}$; which numbers we will

divide by 2, and we have 20 | 10

10 | 5 which is the answer in fact: and as one is the half of two, so is 5 the half of 10; observing that we do not write the denominator.

2. Change $\frac{3}{4}$ to a decimal. Ans. ,75.

4 | 3,00

| ,75 § Read 75 one-hundredths.

3. Change $\frac{4}{5}$ to a decimal. Ans. ,8.

5 | 4,00

| ,8 § Read 8 tenths.

4. Change $\frac{7}{8}$ to a decimal. Ans. ,875.

5. Change $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to a decimal. Ans. ,25.

2 | 1,00

3 | 2

4 | 3

,25

6. Change $\frac{5}{6}$ to a decimal. Ans. ,8333+.

CASE SECOND.

To change a decimal to a vulgar fraction.

EXAMPLES.

1. Change ,75 to a vulgar fraction. Ans. $\frac{3}{4}$.

4 100 | 75 3

4 | 3

2. Change ,875 to a vulgar fraction. Ans. $\frac{7}{8}$.

1000 | 875

8 | 7

To multiply decimals.

§ Multiply the same as in whole numbers; observing to cut off as many figures in the product as there are decimals in both the multiplier and multiplicand.

EXAMPLES.

1. Multiply 4,85 by ,42. Ans. 2,0370.

4,85	or,	100	485
,42		100	42
			970
			1940
		1,0000	2,0370

2. Multiply 2,5 by 3,5. Ans. 8,75.

2 10	25 5	2 10	25
2 10	35	2 10	35 7
	8,75		$8\frac{3}{4}$

3. Multiply 4,62 by 3,25. Ans. 15,015.

DIVISION BY DECIMALS.

EXAMPLES.

1. Divide 12 feet by ,5 of a foot. Ans. 24.

§ Now, as 5 is in tenths, we will reduce 12 to tenths.

Thus: 5 | 10 2 / 12 / — / 24

You will observe that 5 is one-half of 10; hence the divisor is the same as $\frac{1}{2}$, and one-half is in any number of units twice as many times as there are units.

2. Divide 27 by ,3. Ans. 90.

$$\begin{array}{r|l} 3 & 27\ 9 \\ & 10 \\ \hline & 90 \end{array}$$

3. Divide ,55 by 11. Ans. $\frac{1}{100}$ or, 0,05

$$\begin{array}{r|l} 100 & 55 \\ 11 & \\ \hline & 0,05 \end{array}$$

4. Divide 100 by ,25. Ans. 400.

$$\begin{array}{r|l} 25 & 100 \\ & 100\ 4 \\ \hline & 400 \end{array}$$

5. Divide 600 by ,75. Ans. 800.

6. If 350 pounds of beef cost $12,25, what will one pound cost? Ans. 3½ cents; or ,035 of one dollar.

7. At 7 per cent. per annum, how much capital must be invested to yield $602? Ans. 8600.

$$\begin{array}{r|l} 7 & 602\ 86 \\ & 100 \\ \hline & \end{array}$$

8. A benevolent individual gave away 12 per cent. of his income; what was his income if he gave away 600 dollars? Ans. $5000.

9. Change $\frac{1}{2}$ to a single fraction. Ans. $\frac{1}{4}$.

$$\begin{array}{r|l} 2 & 1 \\ 2 & 3 \\ \hline & \frac{1}{4} \end{array}$$

(*Wilson's Arith. p.* 79.)

10. Change $\frac{1}{2}$ to a single fraction. Ans. $\frac{1}{11}$.

(*W.* 79.)

11. Change $\frac{\frac{2}{3}}{\frac{1}{10}}$ to a single fraction. Ans. $\frac{20}{3}$.

(*W.* 79.)

12. Change $\frac{\frac{2}{3}}{\frac{11}{12}}$ to a single fraction. Ans. $\frac{8}{11}$.

(*W.* 79.)

13. Change $\frac{\frac{1}{12}}{\frac{5}{8}}$ to a single fraction. Ans. $\frac{2}{15}$.

(*W.* 79.)

14. Change $\frac{3\frac{1}{2}}{3\frac{3}{4}}$ to a single fraction. Ans. $\frac{14}{15}$.

2 | 7
11 | 3

(*Ray's*, 136.)

15. Reduce $\frac{7\frac{2}{12}}{9\frac{5}{12}}$ of $\frac{2\frac{1}{2}}{3\frac{2}{15}}$ to a single fraction. Ans. $\frac{70}{171}$.

(*R.* 137.)

MISCELLANEOUS EXAMPLES IN REDUCTION, FRACTIONS, &c.

EXAMPLES.

1. Reduce $\frac{1}{64}$ of a shilling to the fraction of a farthing. Ans. $\frac{4}{1}$.

2. Reduce $\frac{5}{1000}$ of a pound (Troy) to the fraction of an ounce. Ans. $\frac{5}{3}$.

3. Reduce $\frac{2}{6433}$ of a hogshead to the fraction of a quart. Ans. $\frac{8}{257}$.

4. Reduce $\frac{1}{7056}$ of an acre to the fraction of a rod. Ans. $\frac{40}{1089}$.

5. Reduce $\frac{3}{4}$ of a farthing to the fraction of a £. Ans. $\frac{1}{1280}$.

6. Reduce $\frac{1}{12}$ of $\frac{1}{3}$ of £4 to the fraction of a penny. Ans. $\frac{80}{3}$.

7. Reduce $\frac{9}{5}$ of $\frac{1}{12}$ of $\frac{1}{3}$ of a penny to the fraction of a £. Ans. $\frac{1}{720}$.

8. $\frac{9}{5}$ of one penny is $\frac{1}{12}$ of $\frac{1}{3}$ of what fraction of four £? Ans.

9. $\frac{7}{60}$ of one £ is $\frac{1}{14}$ of what fraction of seven guineas? Ans.

10. What is the value of $\frac{2}{3}$ of a £?

Ans. 13 and $\frac{1}{3}$ shillings.

$$3 \mid 2$$
$$\quad\; 20$$

$13\frac{1}{3}$ shillings.

11. What is the value of $\frac{5}{8}$ of a shilling.

Ans. $7\frac{1}{2}$ pence.

$$2\ \cancel{8} \mid \begin{matrix} 5 \\ \cancel{12}\ 3 \end{matrix}$$

2 | 15 or $7\frac{1}{2}$ pence.

12. Reduce 7 pence 2 quarters to the fraction of a shilling. Ans. $\frac{5}{8}$ shillings.

$$2\ \cancel{12} \mid \begin{matrix} 4 \\ \cancel{20}\ 5 \end{matrix}$$

$\frac{5}{8}$

13. What is the amount of $\frac{5}{8}$ of a mile?

Ans. 6 fur. 26 rd. 3 yd. 2 feet.

$$\begin{matrix} & 5 \\ & 8 \\ 6 & 40 \\ 2 & 11 \end{matrix}$$

6. 26. 3. 2.

14. What is the value of $\frac{9}{11}$ of a ton?

Ans. 16 cwt. 1 qr. $12\frac{8}{11}$ lbs.

15. What is the decimal of $\frac{1}{3}$ of a month? Ans. ,8.

16. What is the decimal of $\frac{3}{4}$ of a dollar?

Ans. ,75 cts.

17. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to a decimal fraction.

Ans. ,25.

18. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{1}{2}$ divided by $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{1}{6}$, to a decimal fraction. Ans. 2,25.

19. Reduce 1 shilling, 4 pence, 2 qr., to the decimal of a £? Ans. ,0875.

20. If $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{9}$ of a ship be worth $\frac{7}{9}$ of $\frac{5}{7}$ of $\frac{9}{15}$ of the cargo, valued at $36.000, what is the value of the ship?

Ans. $45.000.

21. If one yard of ribbon cost 8 pence, what will 72 yards cost, (New-England currency?) Ans. $8.

	72 ¢
1	8
12	1
¢	1.

8 dollars.

22. If 4 pounds of nails cost 18 pence, how many dollars (New-England currency) will 12 tons cost at the same rate? (*P. B.*) Ans. $1680.

23. If one pint of rum cost 10 pence, what will three hogsheads cost, (New-England currency?) Ans. $210.

	3
	£3 21
	4
3 12	2
20	10
3	10

210

24. If 3 hogsheads cost £63, what will 1 pint cost?

(*P. B.*) Ans. 10 pence.

25. If 18 pence will buy 4 pounds of nails, how many tons can be purchased for $1680, (New-England currency?) (*P. B.*) Ans. 12 tons.

26. If $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{6}$ of 40 men purchase the $\frac{4}{5}$ of $\frac{1}{2}$ of $\frac{7}{8}$ of 8 acres of land, what is each man's share? Ans. $\frac{7}{10}$ acres.

27. Change 1296000 cubic inches to tons of hewn timber? Ans. 15 tons.

28. If 16 compositors set 150 pages of type, each page consisting of 48 lines, and each line of 50 ems (or squares,) in 8 days, 10 hours long; how many compositors will be required to set 500 pages of 72 lines each, and 45 ems in a line, in 6 days, 8 hours long? Ans. 120.

29. If a cellar 22,5 feet long, 17,3 feet wide, and 10,25 feet deep be dug in $2\frac{1}{2}$ days, by 6 men working 12,3 hours a day; in how many days of 8,2 hours would 9 men dig a cellar 56,25 feet long, 25,95 feet wide, and 16,4 feet deep? (*Chase's*, 99.) Ans.

30. If 36 men in $127\frac{1}{2}$ days, of $13\frac{1}{2}$ hours, dig a trench $33\frac{1}{4}$ yds. long, $10\frac{1}{2}$ feet deep, and $15\frac{2}{3}$ wide; how many men in $7\frac{3}{4}$ days of $12\frac{8}{11}$ hours, will dig a similar trench $82\frac{8}{11}$ yds. long, $7\frac{2}{3}$ feet deep, 10 feet wide? (*C.* 99.) Ans.

31. If 19 men in $71\frac{3}{4}$ days of $10\frac{3}{5}$ hours, dig a trench $41\frac{1}{2}$ yds. long, $5\frac{7}{8}$ feet deep, and $7\frac{1}{3}$ feet wide; how long a trench that is $8\frac{2}{3}$ feet deep, and $4\frac{7}{10}$ wide, will 11 men dig in $291\frac{1}{2}$ days, of $4\frac{7}{9}$ hours long? (*C.* 99.) Ans.

32. How many days of $8\frac{1}{2}$ hours will 42 men require to build a wall $98\frac{3}{4}$ feet long, $7\frac{1}{2}$ feet high, and $2\frac{3}{4}$ feet thick, if 63 men can build a wall $45\frac{1}{4}$ feet long, $6\frac{7}{12}$ feet high, and $3\frac{1}{3}$ thick, in 68 days of $11\frac{1}{2}$ hours? (*C.* 99.) Ans.

33. A gentleman has a carriage worth $500, and two valuable horses. If the first horse be harnessed to the carriage, then the horse and carriage is worth three times as much as the second horse. But if the second horse is harnessed to the carriage, then the second horse and carriage

are worth 7 times as much as the first horse. What is the value of the horses?

Ans. { First horse, $100. Second do. $200.

34. If 240 men, in $5\frac{1}{2}$ days of 11 hours, dig a trench $232\frac{1}{2}$ rods long, $3\frac{2}{3}$ feet wide, and $2\frac{1}{2}$ feet deep; in how many days of 9 hours, will 24 men dig a trench $337\frac{1}{2}$ rods long, $5\frac{3}{5}$ feet wide, and $3\frac{1}{2}$ feet deep, the ground being in the latter case but $\frac{4}{5}$ as hard as the former?

36. A. B. and C. meeting on the road, agreed to dine together. A. furnishes 5 loaves, B. 3, and C. having no bread, pays 8 equal pieces of money for his share. How should the money be divided between A. and B.?

Ans. B. 1 piece, A. 7 pieces.

37. If 12 oxen in 4 weeks eat $3\frac{1}{3}$ acres of grass, and 21 oxen eat 10 acres of the like pasture in 9 weeks, how many oxen will eat 24 acres in 18 weeks, the grass being at first equal on every acre and growing uniformly?

(*Newton's Universal Arith.*) Ans. $37\frac{11}{17}\frac{3}{3}$ oxen.

RATIO AND PROPORTION.

REMARKS.

§ It may be said by those who do not understand the principle on which questions are worked by this rule, that it will solve very few of the questions that arise in business; but this conclusion will be found incorrect upon examination. For as this rule is based on the only principles that belong to numbers, that of increase and decrease, it is evident that it will solve all questions that can arise.

Also, the principle of canceling applies to almost all questions that occur in business; but it must be remembered that though we do not cancel a single figure, yet the rule is not at all infringed; for we get the same result by decreasing those on the right by those on the left.

§ The rule is Increase and Decrease, and this principle (as before remarked) variously applied, will solve every question that can arise, that admits of solution.

It is entirely distinct from canceling, for canceling is only the application of this principle to particular cases; and although, as has been remarked, canceling will apply to at least seventy-five out of each hundred questions that would arise in the first one hundred, according to the theory of numbers, and a much larger proportion as we ascend above this, and in actual business will solve nearly every case that occurs: yet it must be remembered that it is not the rule.

It is true that in Arithmetic, we may find questions like this: What will 29 yds. 3 qr. 3 na. of cloth cost, at £2, 5s. 3d. 1 farthing per yd.

Now let me inquire, what would you think of an individual who would ask a shop-keeper, in sincerity, for 6 pounds, 11 ounces, 5 drachms of tea. And suppose the shop-keeper should charge him 5 shillings 7 pence 3 farthings per pound for it; would you not think them both insane, and fit subjects for a strait jacket; and yet we find many of the questions in our Arithmetics that have the same amount of good sense. It may be asked if I would not have the scholar able to do such sums? I would first give him a thorough knowledge of the principles of numbers, and that too as applied to the business calculations of the day; and then if he has no better employment, he may wade through all the examples that are placed in our Arithmetics for the purpose of making a book.

Let me ask why it is that business men seldom or never use the rules they have learned in Arithmetics? Is it not owing to the fact that those rules are not applicable to business transactions.

§ I shall now present a principle by which all questions in Proportion can be stated; and this method can be used for arranging the questions before placing them on the line, till the principle is perfectly familiar, and then like a scaf-

fold used in erecting a building, it may be thrown away as useless.

PRINCIPLE.

In Philosophy we find the following axioms; to wit:

Equal causes produce equal effects.
Like causes produce like effects.
Effects are always proportioned to their causes.
An effect was never produced without a cause.

Keeping these facts in mind, we form the following rule, viz: that any given *cause* is to its *effect* as any required *cause* is to its *effect;* or, any given *effect* is to its *cause* as any required *effect* is to its cause.

The only difficulty that can arise in this method of statement, is to determine what is cause and what is effect; but this difficulty must vanish when we remember that all action is cause, and the result of action is effect.*

EXAMPLES.

1. If 36 men build a wall in 16 days, in what time will 12 men build a like wall, working at the same rate?

Cause.		*Cause.*		*Effect.*		*Effect.*
36	:	12	: :	1	:	1
16	:	x				

§ In the example, we have the two *effects* and one *cause* and part of the other given; and it is evident in examining the question, that 36 men in 16 days is the first *cause;* and the wall they built the first *effect:* and 12 men is part of the second *cause;* and the wall they build is the second *effect.* As part of the second *cause* is blank, we use x to fill the place. Now the extremes are always multiplied together, and also the means. We write the factors under one

* *Causes* in computation are men, horses, time, capital, money lent, &c., &c. That which is done is effect, as work performed, grain consumed, wheat ground, loss sustained, distance traveled, &c., &c.

another, as in the example; and as part of the second cause is an unknown term, we have used x to represent it. If all of the second *cause* was known, then the product of the extremes would equal the product of the means, and we should have a perfect proportion.

§ In the example given, we draw the line and place the means on the left and the extremes on the right.

Thus:

12 |	36
1 |	1
x |	16
x |	48 Ans.

And then cancel as in the rule, and we find that x=48.

2. If $100 in 12 months gain $6, what will $1600 gain in 8 months?

§ In this example the money and time is the *cause*, and the interest the *effect*.

We use in Algebra the last letters of the Alphabet, to represent the terms that are not known. For example: I wish to know what the interest of 400 dollars is for 6 months, at 6 per cent. per annum. It is supposed that I know the amount of $100 for one year. Now I let x represent the interest of $400 for 6 months, and then I say if $100 in 12 months gain $6, then $400 in 6 months at the same rate will gain x. Then the statement will stand thus:

C. *C.* *E.* *E.*

100 : 400 :: 6 : x

12 6

Or, it will stand thus:

100 |	400
12 |	6
x |	6
x |	12

§ The blank or x, always falls on the left. That is, the product of the two outside terms must equal the product of the middle terms. Hence, if the blank which we call x falls under either of the middle terms (which are called the means,) they are placed on the left. Solving the question, we find that x

equals 12, and as the answer was to be in dollars, then 12=the answer in dollars.

§ We will now suppose as an example, that $12 had been received as the interest of $400 at 6 per cent., and that the length of time was not known; then the x would be used to represent the time, and then as the blank or x falls under one of the middle terms in the example, they will be placed on the left, and from this statement we get $6=x$.

C.	*C.*	*E.*	*E.*
100	400	6	12
12	x		

~~400~~	~~100~~
x	~~12~~ $
~~2~~ ¢	~~12~~ 6
x	6 Ans.

§ Again, suppose that I had received $12 as interest on an unknown sum for 6 months, and wished to know the principal. I then should make the following statement:

C.	*C.*	*E.*	*E.*
100	x	6	12
12	6		

x	100
6	12
6	12
x	400 Ans.

§ Now if I had received $12 as the interest on $400 for 6 months, and wished to get the rate per cent.; the statement then would be as follows:

C.	*C.*	*E.*	*E.*
100	400	x	12
12	6		

400	100
6	12
x	12
x	6 Ans.

EXAMPLES.

1. What is the interest of $450 for 2 years, at 5 per cent.?

C.		C.		E.		E.
100	:	450	: :	5	:	x
12	:	24				

$\not{2}\ \not{100} \mid 45\not{0}$
$\not{12} \mid \not{5}$
$x \mid \not{24}$

x | 45 dollars, Ans.

2. What is the interest of $560 for 2 years and 6 months, at five per cent? (*Talbot* 114.) Ans. $70.

3. What principal at interest for 8 years, at 5 per cent., will amount to $840? (*T.* 116.) Ans. $600.

C.		C.		E.		E.
140	:	840	: :	100	:	x

$\not{140} \mid \not{84}\not{0}$ 6
$x \mid 100$

§ Find the amount of $100 for the given rate and time.

x | 600 Ans.

4. What principal at interest for 6 years, will amount to $1240, at 4 per cent. per annum? (*T.* 117.) Ans. $1000.

5. What principal at interest for 5 years, at 6 per cent. will amount to $2470. (*T.* 117.) Ans. $1900.

6. At what rate per cent. per annum will $600 amount to $744, in 4 years? Ans. 6 per cent.

NOTE.—Subtract the principal from the interest.

C.		C.		E.		E.
600	:	100	: :	144		x

$\not{6}\not{0}\not{0} \mid \not{1}\not{0}\not{0}$
$\not{4} \mid \not{144}$
$x \mid 1$

x | 6 Ans.

7. At what rate per cent. will $1200 amount to $1476, in 5 years and 9 months? Ans. 4 per cent.

8. In what time will $400 amount to $520, at 5 per cent.? (*T.* 118.) Ans. 6 years.

NOTE.—Find the interest of the principal for one year.

C. *C.* *E.* *E.*
20 : 120 :: 1 : *x*

20	120
x	1

x | 6 Ans.

COMPOUND INTEREST.

1. What is the amount of $1300 for 3 years, at 5 per cent., compound interest? (*T.* 119.) Ans. $1504,91+.

x	
1 0 0	1 3 0 0
100	1 0 5 21
20 1 0 0	1 0 5 21
4 2 0 1 0 0	1 0 5 21
8*x*	12039,30
	1504,9125

2. What is the amount of $1000, at 10 per cent. compound interest, for 4 years? Ans. $1464,10.

MISCELLANEOUS EXAMPLES.

1. What is the commission on $1320, at 5 per cent.? Ans. $66.

2 1 0 0	1 3 2 0
x	$
x	66

2. What is the brocage on $2150, at 2 per cent. (*T.* 121.) Ans. $43.

3. Bought 126 gallons of wine for $150, and retailed the same for 20 cents per pint; what was the gain per cent?

Ans. $34,40 or 34,4 per cent.

5 150 | 126 42
20
x | 2
4

| 134,40 Ans.

4. If $400 gain $18 in 9 months, what was the rate per cent. per annum.—(*Slocum's*, 73.)

	C.	C.		E.	E.
400 | 100	400	: 100	: :	18	: x.
9 | 12 3		9		12	
x | 18 6					

x | 6 Ans.

5. If a usurer put out $75 at interest, and at the end of 8 months receives for principal and interest $79, at what rate per cent. did he receive interest? Ans. 8 per ct.

$ 75 | 100 4 (S. 73.)
12 4
2 8 | 4 2

8 Ans.

6. If 7 men can reap 84 acres of grain in 12 days, in how many days will 20 men reap 100 acres.

(S. 73.) Ans. 5 days.

7. What principal at 6 per cent will gain $14 in 7 months? (S. 73.) Ans. $400.

8. What is the interest of $313,06 for 1 year 11 months, at 6 per cent? (*Smith's*. 173.) Ans. $36,00.

$13,06 156,53
12 | ¢
23

$36,00

9. What is the interest of £500 for 2 years 1 month?

12	$$$ 250
	$
	25

£62,50 Ans.

10. A bell of Moscow weighs 288.000 lbs., how many tons? (*R.* 93.) 128¼ Ans.

2000	2000000 288000 900
4	
7 2000	

128¼ tons.

11. How many coat patterns, each containing 3 yds. 2 qrs. can be cut, out of a piece of cloth 7 ells (Flemish) in length? 2 1/4 | 7000 1000 5 Ans. 15.

4	4
	3

15

12. If, when wheat is 83 cents per bushel, the cent loaf weighs 9 ounces, what ought it to weigh when wheat is $1,24½ per bushel? (*T.* 97.) Ans. 6.

x	$ 3	*C.*	*C.*		*E.*	*E.*
$3	83	: 1,24½	: :	*x*	: 9
3 249	2					

6 Ans.

13. If 100 dollars principal in 12 months gain 6 dollars interest, what principal will gain the same interest in 8 months? (*T.* 97.) Ans. $150.

C.	*C.*		*E.*	*E.*		*C.*	*C.*	*E.*	*E.*
100	: *x*	: :	8	: 12	or,	100	: *x*		
						12	: 8	: : 6	: 6

§ Capital will not produce interest without time; and from that fact you will conclude that time, as well as the principal, is cause. *See second statement.*

14. There is a cistern having a pipe that will discharge the water in it in 12 hours; how many pipes of the same capacity will discharge it in 15 minutes? Ans. 48.

x	1
$I\$$	12*
$\phi\phi$	4

(*T.* 97.)

C. C. E. E.
1 : x :: 15 : 12
60

48

§ * Reduce the 12 hours to minutes.

15. If 7 pounds of cheese cost 87½ cents, what must I pay for 122 pounds? (*T.* 95.) Ans. $15¼

$	7
7	122

C. C. E. E.
7 : 122 :: 87½ : x

15¼

16. If 15 yards of broad cloth cost $80, what will 75 cost? (*T.* 95.) Ans. $400.

17. If 321 bushels of salt cost $240,75, what will one bushel cost? Ans. 75 cents.

18. What do 50 firkins of butter cost, at $7,14 per firkin? (*D.* 31.) Ans. $357.

2	7,14
	1

19. How many pieces of Holland may I have, each piece 20 ells Flemish, for £ 23, 8 shillings, at 6 s. and 6 pence per ell, English? (*D.* 96.) Ans. 6.

$	$II7\ 3\phi\ 3$
$I3$	$4\phi\ 2$
2ϕ	$
3	
	6

£. s.
23 8 = $^{1}1\frac{7}{5}$ £.
6s 6d = $\frac{1}{40}\frac{3}{0}$ £.

20. Bought a pipe of wine for $84, but found it had leaked out 12 gallons; I sold the remainder for 12½ cents per pint; did I gain or loose, and how much? (*L.* 96.) Gained $30.

21. A school master being asked how many scholars he had, answered, that if he had as many more as he then had, half as many more, one third as many more, and one-fourth as many more, he then would have 148; how many scholars had he? (*D.* 189.) Ans. 48.

§ Let x represent the school. Then, according to the master's reply, he must have x, or another school added to the one he then had; this would make him $2x$, or two schools the size of the one he had. He also says that he must have one-half as many more, one-third and one-fourth, to make 148.

These will then be $\frac{1}{2}$ of x, $\frac{1}{3}$ of x, $\frac{1}{4}$ of x, and may be represented thus: $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}$; but before we can add these, it will be necessary to bring them to a common denominator. The least common denominator is 12, and $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{13}{12}$; and then, taking $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ of x, it will equal $\frac{13}{12}x$; to which we must add the two schools, and we have $2\frac{13}{12}$; or, $2\frac{13}{12}x=3\frac{1}{12}x$.

37 | 148 4
x | 12

| 48 Ans.

Thus we find that the school=48.

22. What is that number which being increased by $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of itself, the sum will be 125. Ans. 60.

(*D.* 189.)

§ $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{25}{12}$ of 1; or let x be the representative of the number sought; then we have x=the number we wish to find, and we have $x+\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=\frac{25}{12}x$.

25 | 125
x | 12

x= 60 Ans.

Then as the number sought can only be $\frac{12}{12}$ or one, we place 125, (the given number,) and 12, on the right, and 25 and x on the left; then we find that 25 cancels 125 on the right, and leaves 5 and 12, which are multiplied together for the answer, and we have 60=x the answer.

23. A purse of $160 is to be divided between A. B. C. and D. so that B. may have 4 dollars more than A., and C.

8 dollars more than B., and D. twice as much as C.; what is the share of each man? (*D.* 190.)

§ As A. has one share, we may call it 1, or x.

Then $x=1$ or A.'s share;
and B. had $4 more, $1+x=1+4$ B.'s "
and C. $8 more than B., $8+4+x=1+4+8$ C.'s "
and D. twice C.'s share, or $16+8+2x=2+8+16$ D.'s "

We will now add $24+16+5x=5$ shares, and $40 more, $24+16+5x=5x+40$, and this is equal to $100.

$5x+40$ |	100 Divide by 5.
$x+8$ |	20 Subtract 8 from both sides, and $x=12$ or, A.'s share, and $12+4=16$ or B.'s share.

$x=12$ or A.'s share.
$4+x=16$ " B.'s "
$8+4+x=24$ " C.'s "
$16+8+4+x=48$ " D.'s "

24. A laborer was hired 60 days, upon these conditions: For every day he worked, he should receive 4 shillings, and for every day he was idle, he should forfeit 2 shillings. At the expiration of the time, he received £7 10 *s.* How many days did he work, and how many was he idle? (*D.* 191.) Ans. 45 he w. 15 days id.

§ As the number of days he worked or was idle is unknown, we will call it x; then the days he worked will be the difference between this and 60.

Let x=days he was idle,
and $60-x$=the days he worked.

Then four times the days he worked will be the number representing the money he received; and twice the days he was idle will represent the sum he must pay back; and this subtracted from the money he received, will leave 7 pounds 10 shillings.

4 times $60-x=240-4x$
2 " $x = 2x$

Then $240-4x-2x=$ | 150 — 7 £ 10 s.=150 s.

$90 \quad 6x$ | or, $\not{6}x \mid \not{9}\not{0}$

x | 15 days he was idle.

Drop 150 from both sides, and $2x$ and $4x=6x$. Then place 90 on the right, and let $6x$ stand on the left; change the—numbers to plus; then divide by 6, and $x=15$, the days he was idle, and $60-15=45$, the days he worked.

25. There are three horses belonging to three men employed to draw a load of plaster from Boston to Windsor, for $26,45. A's and B's horses are supposed to do $\frac{3}{4}$ of the work; A's and C's $\frac{9}{10}$; B's and C's $\frac{13}{20}$; they are to be paid proportionally; what is each man's share of the money? (*Adams'* 175.)

2nd, A+B's=$\frac{15}{20}$ — $\frac{3}{4}=\frac{15}{20}$
1st, A+C's=$\frac{18}{20}$ — $\frac{9}{10}=\frac{18}{20}$
1st, B+C's=$\frac{13}{20}$ — $\frac{13}{20}=\frac{13}{20}$

2nd, A—B=$\frac{5}{20}$ Then add this to A. and B's.

$2A=\frac{40}{20}$ Or one A=$\frac{18}{20}$.

A's, | 10
B's, | 5
C's, | 8

$\not{2}\not{3}$ | +10+5+8
$\not{2}\not{6},\not{4}\not{5}$ 11,5

11,50=A's
5,75=B's
9,20=C's

26,45 the sum.

26. A person after spending $\frac{1}{2}$ and $\frac{1}{3}$ of his income, has $30 left. What had he at first?

(*B.* 215.) $\frac{1}{2}$ and $\frac{1}{3}=\frac{5}{6}$. Let x=his income.

Then he spent $\frac{5}{6}$ of it, and $30 must be the other $\frac{1}{6}$ of it; hence, we had 6 times 30 or 180 dollars.

1	6
x	30

180 Ans.

27. Three men are employed to draw a load of salt from Boston to Lowel, for $9,50. A. and B's horses are supposed to do $\frac{7}{8}$ of the work; A. and C's $\frac{7}{12}$; B. and C's $\frac{17}{24}$; they are to be paid proportionally. What is each man's share? (*B.* 245.)

A+B's	21
A+C's	14
B+C's	17
B—A	3
2 B	24
B	12
C	5
A	9

$\frac{7}{8}=\frac{21}{24}$
$\frac{7}{12}=\frac{14}{24}$
$\frac{17}{24}=\frac{17}{24}$

2ϕ	+12+5+9
$\$, \$\phi$	$36\frac{7}{12}$

$4,38\frac{6}{12}$=B's
$1,82\frac{9}{12}$=C's
$3,28\frac{1}{12}$=A's.

28. A sum of money is to be divided between 5 men, in such a manner that A. shall have $\frac{1}{4}$, B. $\frac{1}{5}$, C. $\frac{1}{10}$, D. $\frac{1}{20}$, and E. the remainder, which is $40; what is the sum?

(*B.* 216.) Ans. $100.

29. What will one hogshead of wine cost, at 6¼ cents a gill? (*R.* 95.) Ans. $126.

	63
	4
	2
100	4
4	25
	126

30. If a person write 10 minutes per day, what will it amount to in four years? (*R.* 96.) Ans. 10d. 3½h.

	365
60	4
6 24	10

$10\frac{5}{6}$ Or 10 days 3½ hours.

31. How many yds. of carpeting 2 feet 6 inches wide, will it take to cover a floor 27 feet long, and 20 feet wide? (*T.* 98.) Ans. 72.

32. How many men will it take to build a wall 75 rods long, 8 feet high, 3 feet thick, in 6 days, working 9 hours each day, if 20 men can build a wall, 100 rods long, 6 feet high, and 4 feet thick, in 12 days, working 12 hours each day? (*B.* 136.) Ans. 40 men.

33. If ¾ of a yard of cloth cost £⅔, being ⅞ wide, what is the value of ⅝ of a yard 1¾ wide, of the same quality? (*B.* 136.) Ans. £⅔.

34. Bought 126 gallons of wine for $110; how much water must be added to reduce the first cost to 75 cents per gallon. (*P.* 176.) Ans. 20⅔.

	110,00	146⅔	
110	120		146⅔
75			—126
		146⅔	20⅔

35. A section of land is a mile square; suppose a circle be inscribed in one touching all the sides, what quantity of land would be in the section and not in the circle?
(*P.* 177.) Ans. 137,36 acres.

36. What will a bushel of clover seed cost, at $12\frac{1}{2}$ cents per pint? (*W.* 41.) Ans. $8.

	4 pecks
	ꝑ qrt.
	2 pt.
ꝑ	1 $12\frac{1}{2}=\frac{1}{8}$ of $1

8 Ans.

37. Suppose that a hogshead of molasses which cost $23, be retailed at $12\frac{1}{2}$ cents a quart, what will be the profit on it? (*W.* 41.) Ans. $8,50.

38. What will 5 barrels of flour cost, at $3\frac{1}{2}$ cents per pound? (*Ru.*) Ans. $34,30

39. If one quart of vinegar cost 8 pence, how many £ will five hogsheads cost? (*Po.* 21.) Ans. £42.

	ꝑ 2d.
	ꝑ hhd.
	ꝑꝑ 21 gal.
	₰ qt.
12	1
20	1

£42

40. If 4 pounds of nails cost 18 pence, how many £ (New-England currency) will 12 tons cost, at the same rate? (*Po.* 21.) Ans. £450.

41. If 8 gallons of N. E. Rum cost $4, how many £ (New York currency) will 15 pipes cost?
(*P. O.* 22.) Ans. £378.

42. If 4 pounds of nails cost 18 pence, how many dollars (New-England currency) will 12 tons cost, at the same rate? Ans. $1680.

	£ 6
£	£
£	£
£	£
	28
£	10
	1680

43. There is a fish whose head is 3 feet long; his tail is as long as his head and half the length of his body, and his body is as long as his head and tail both. What is the length of the fish? (*B.* 218.) Ans. 24 ft.

44. A man meeting a boy driving a flock of geese, said: "Good morning, sir, with your 100 geese," Says the boy: "I have not a hundred; but if I had as many more as I now have, half as many more, and $2\frac{1}{2}$ geese, I should have 100." How many had he?

(*Parks. &c.*, 175.) Ans. 39 geese.

$100-1+1+\frac{1}{2}=100-2\frac{1}{2}=97\frac{1}{2}$

$\frac{5}{2}$	2
2	195
	39 Ans.

45. How many shingles, each covering 24 square inches, upon an average, will it take to cover a house 40 feet in length, and 18 feet rafters? (*P.* 174.)

Ans. 8640 shingles.

46. How many bricks, each 10 inches long, 5 inches wide, and 3 inches thick, will it take to build a wall 10 feet high, 500 feet long, and 15 inches thick?

(*P.* 174.) Ans. 72000 bricks.

37. The paving of a square, at 2 shillings per square yard, costs as much as inclosing it at 5 shillings. What is the length of one of the sides? Ans. 10 yds.

48. How many bricks will it take to build a square pyramid, the base of which is 100 feet, and running to a point 100 feet high? (*P.* 174.) Ans. 3840000 bricks.

10	100 2
5	100
3	100
3	172 1920000

3840000 Ans.

49. A hopper is 6 feet square at the top, and 5 feet deep, and is an inverted cone; what number of bushels will it hold? (*Parks*, 174.) Ans. 48+ bush.

	6 3
	6
3	5
14 50	45

675 (48+)

50. Prove that a garner 8 feet long, 4 feet wide, and 4 feet 6 inches deep, will hold 115,7+ bushels.

51. What is the difference between a floor 48 feet long and 30 feet wide, and that of two others, each one half the dimensions? (*B.* 222.)

Ans. the first is twice as large.

52. A roof 18 feet 8 inches by 14 feet 6 inches, is to be covered with lead at 8 pounds per foot. What will it cost at 18s. per cwt.? (*B.* 222.) Ans. £22 19+s.

53. A cable which is 3 feet long and 9 inches in compass, weighs 22 pounds; what will a fathom of that cable weigh, that is 9 inches in diameter?

(*B.* 224.) Ans. 434¼ lbs.

54. A circular fish-pond is to be dug in a garden, that shall take up just one half an acre of ground; what must be the length of that chord that will strike the circle?
(*B.* 224.) Ans. 27¾ yds.

	4
2	40
2	11
2	11
x^2	
3,1416	

55. A carpenter is to put an oaken curb to a round well, at 8 pence per foot square; the breadth is to be 7¼ inches, and the diameter within 3½ feet; what will be the expense?
(*B.* 225.) Ans 5s. 2½d.

56. Suppose the expense of paving a semi-circular plot at 2s. 4d. per foot, amounts to £10; what is the diameter of it? (*B.* 225.) Ans. 14,7739 ft.

57. Seven men bought a grindstone of 60 inches diameter, each paying ⅐ of the expense; what part of the stone must each man grind down for his share, commencing with the first, and so on alternately?
(*B.* 225.)

x	60
x	60
3,1416	3,1416
7	1
x^2	514,2857

Or, x | 22,6778

Ans. The 1st, 4,4508 in.
2nd, 4,84+ "
3d, 5,3535 "
4th, 6,0765 "
5th, 7,2079 "
6th, 9,3935 "
7th, 22,6778 "

ROOTS.

SQUARE AND CUBE ROOTS.

To extract the Square or Cube Root.

§ To extract the roots with facility, it is necessary to be well acquainted with the following properties of numbers:

I. *That a square number multiplied by a square, the product is a square.*

II. *That a square divided by a square, the quotient is a square.*

III. *That a cube number multiplied by a cube, the product is a cube.*

IV. *That a cube number divided by a cube, the quotient is a cube.*

V. *That a composite number is the product of two or more numbers, and may be divided by those numbers without a remainder.*

VI. *That a square or cube root of any number, is a composite number, the square or cube of which, may be divided into intiger square or cubic factors.*

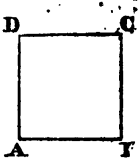
VII. *That the square or cube root of a prime number, cannot be separated into even factors; that is, the factors will involve fractions, seme of which are indeterminate.*

§ It will be necessary for those who wish to extract the cube or square roots with any thing like skill and facility, to be able to recognise the squares and cubes of the first 9 digits, at first sight; and for that purpose the following table is inserted:

NUMBERS,	1	2	3	4	5	6	7	8	9	10
SQUARES,	1	4	9	16	25	36	49	64	81	100
CUBES,	1	8	27	64	125	216	343	512	729	1000

The reader will not forget that this is an Appendix, and it is supposed that he is already acquainted with the old method of extracting the roots.

VARIOUS METHODS OF EXTRACTING ROOTS.



A Square,

Is a figure bounded by four equal sides; and extracting the Square Root consists in finding the length of the side of the square.

1. What is the square root of 1225?

3	1225(35	§ Look at the table after pointing the figures off into periods of two figures each, and find the greatest square root of 12, which is 3; place this as the first figure of the root, also place it at the left of the perpendicular line; then multiply 3 by 3, and subtract the product from 12; then bring down the left hand figure of the next period; add the threes together. Then use this as a divisor, and see how many times it is contained in the remainder, with the left hand figure of the next period, and place the result as the second quotient figure; thus continue through the periods, observing to place the quotient figures at the right also of the trial divisors.
3	9	
—	—	
65	32,5	
	32,5	
	—	
	0	

Second:

| 1225(35

§ By looking at the table, we find that 3 is the greatest root of 12, and as the next period ends with 5, the next figure in the root is 5. With the same facility can the root of all squares or cubes that are not primes, be extracted.

2. What is the square root of 729? Ans. 27.

(*S. C.* 116.)

2 | 729(27
4 | 4
——
32

§ By inspecting the table, we find that the first quotient figure is 2; and as the last may by the table be 3 or 7, we subtract the square of the first figure, from the 7 or first period; then to the remainder we bring down the first figure of the next period, and then we try how often the square of the quotient figure is contained in these figures, which we find to be 8 times precisely; from that it is evident that the next figure of the root is 7.

3. What is the square root of 625? Ans. 25.

2 | 625(25 (*S. L.* 117.)

§ By inspecting the table, we find that the first figure is 2; and as 5 is the only figure in the table that has produced 5, we take it for the second figure of the answer.

4. What is the square root of 6561?

(*Ray*, 220.) Ans. 81.

8 | 6561(81
16 | 64
——
1.6

5. What is the square root of 43264? Ans. 208.

(*S.* 119.)

2 | 43264(208
4
——
408 | 03264

§ By inspecting the table, we find 8 to be the last figureof the root, and two to be the first.

6. What is the square root of 106729? Ans. 327.

(T. 170)

3 |	106729(327
6 |	9
	16

§ As the unit figure in this example is 9, if the number is a square, it may be divided either by 9 or 49; after dividing by 9, we have 11881 for the other factor, hence the root is a prime number.

9 |	106729
	11881)109
	1
209 |	01881

§ And the root of 11881 is 109. which multiplied by 3 the root of 9, will give 327 for the answer.

7. What is the square root of 451584? Ans. 672.

4 |	451584
36 |	112896
6 |	18816
16 |	3136
4 |	784
4 |	196
	49 (7×2×4×6×2=672

§ As the unit fig- is 4, and we find in the table only 4 and 64, we will divide 451584 by 4, and we have the factors 4 and 112896. As the last factor closes with 6, we will divide by 16 or 36, which we find in the table; we now have by dividing by 36, 3136; and as this ends with 6, we will divide by 16, and we have for factors 16 and 196. This last factor 196, we will divide by 4, and we have the factors 4 and 49.

We will now extract the roots of the factors

$\sqrt{49\times4\times16\times36\times4}=7\times2\times4\times6\times2=672.$ Ans. 346.

8. Extract the square root of 119716. Ans.

(*B.* 192.)

16	119716		119716(346
4	29929		9
			29
			24

9. What is the square root of 1444? Ans. 38.

(*B.* 192.)

4	1444	
	361(19	§ As the sum ends in 4, the answer must have 8 or 1, for the last figure of the root. First we will divide by 4, and then we have 4 and 261 for the factors, and as the last number ends in 1, the root must be 1 or 9, and the highest root of three is 1, we subtract it from 3, and to the remainder bring down the first figure of the period, and double the one that we have for the first figure of the root, and by inspection we conclude that 9 must be the next figure in the root.
	1	

10. What is the square root of 59536? Ans. 244.

(*B.* 192.)

	59536(244	
2	2	
44		
48	195	
	176	
	193	

11. What is the square root of 2025? Ans. 45.

(*E.* 163.)

25	2025	First divide by 25 and we have 81 and 25 for factors, and by inspecting them, we find that their roots are 5 and 9, equal to 45, the ans.
5	405	
	81(9	

12. Extract the square root of 390625. Ans. 625.
(*Ray's.* 220.)

| 390625
| 4
| 1562500
| 4
| 6250000
| 4
| 25000000(8)5000
| 625

As the sum ends in 5, we can multiply by 4; and as it still ends in 4, we multiply again and again, and as we now have 25,00,00,00, we find that the square root of 25 is 5, and we then add a cipher for every period of the root, and then divide by the product of the roots of the fours.

12 | 390625(625
36
30

13. How many rows on one side of a square cornfield containing 15376 hills? (*B.* 193.) Ans. 492.

22 | 15376(124
1
14 | 53
44
96

or,

16 | 15376
4 | 3844
961(31
4
124

As the sum ends in 6 we can divide by 16, and we have 916 for one of the factors, the square root of which is 31, and and the square root of 16 is 4, and the product of these two roots is 124.

14. An army of 242024 men are placed in a solid body, in the form of a square. What is the number on one of its outer columns? (*B.* 193.) Ans. 482.

| 242024(492
16
8 | 82

We find that 4 is the first figure, and that squared and its product subtracted from 24 leaves 8; bring down 2, and then double the 4, and we find that the next figure will be 9, and the last 2.

15. A man has 841 peach trees that he wishes to plant in a square how many can he plant in each of the outside rows? (*B.* 193.) Ans. 29.

2 | 841(29
4 | 4

44

16. What are two mean proportionals between 4 and 256? (*B.* 200.) Ans. 16 and 64.

16 | 256
---
4 | 64
---
1 | 16(4×4=16 · 4 : 16 : : 64 : 256

17. Find a mean proportional between 4 and 256? Ans. 32.

8 | 256

32

18. What is a mean proportional between 4 and 196? (*T.* 222.) Ans. 28.

4 | 196

49(7×4=28

19. What is the mean proportional between 25 and 81? (*T.* 222.) Ans. 45.

20. There is a square pavement of bricks, consisting of 484 square flag bricks; how many on each side? (*C.* 280.) Ans. 22.

4 | 484
---
11 | 121(11×2=22

21. What must be the area of a square field, that shall contain as much land as a rectangular piece that is 9 by 144 rods? (*C.* 280.) Ans. 36.

36 | 144

4×9=36

22. What is the length in rods of a section of western land containing 640 acres? (*C. W.*) Ans. 320.

$ | ¢404
2 | 4
20 | 400 20

Remove one of the ciphers to 40 or to 640, and then we have all square numbers.

23. A. has two fields, one containing 10 the other 12½ acres; how many rods in length will be the side of a field containing as many acres as both of them? Ans. 60.

1st. *2d.*

10 | 2 | 25 | 10
160 | | 160 | 12½

2000 or, 22½=45
1600 2 | 45 | √900=30
160 | √4=2

60 | 3600(60

60

§ In the second solution we add 10 and 12½, and then reduce them to halves; and then as it makes no difference to which of the numbers the cipher is attached, we place it to the 45, and then we halve the 16 and double the 45, and extract their roots, and the product of the roots gives the answer.

24. The diameter of a circle is 6 inches; what is the size of one 4 times as large? Ans. 12 inches. (*B.* 193.)

6 | 6
6 | 6
2 | 4 2

12

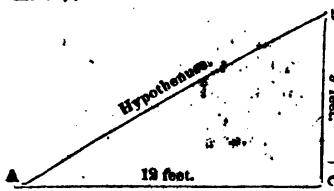
25. The diameter of a circle is 24 feet; what is the area of one only one-fourth as large? Ans. 12 ft di.

6	24 12
2	24
4	1

§ Divide 24 by 6, and multiply 6 by 6, and then we have 4 and 36; their roots are 6 and 2, and the product of 6 and 2 is 12. Circles are to one another as the squares of their diameters.

26. In the right angled triangle A. B. C., the side A. C. is 9 feet and the side B. C. 12; what is the length of the side A. B.?

Ans. 15ft.



3	$+12^2=4^2=16$
3	$+9^2=3^2=9$
	$\sqrt{25}=5$
	3
	15

27. What is a mean proportional between 24 and 96? (*B. T.*)

24	96 4

$2\times24=48$ Ans. 48.

28. What is a mean proportional between 18 and 32?

18	32
$3=9$	$16=4\times3=12\times2$

29. What is a mean proportional between 9 and 81? (*B.* 195.) Ans. 27.

9	81 9

$3\times9=27$

30. The top of a castle is 45 yards high, and is surrounded with a ditch 60 yards wide; what is the length of a ladder that will reach from the outside of the ditch to the top of the wall? (*G.* 128.) Ans. 75 yds.

15	$+45^2=+3^2$
15	$+60^2=+4^2$

$\sqrt{25}=5\times15=75$

31. Two brothers left their father's house; one went 48 miles due west, the other 64 due south, and purchased farms; how far are their farms from each other?
(*T.* 223.) Ans. 80 miles.

10 | 64² 4²=16
10 | 48² 3²= 9

5)25(5×16=80

32. In a right angled triangle the base is 40, and the perpendicular 30 feet; what is the length of the hypothenuse? (*T.* 121.) Ans. 50 feet.

10 | 30 3
10 | 30 3
9

10 | 40 4
10 | 40 4
16+9=25

√25=5 5×10=50

33. The walls of a fortress are 36 feet high, and the ditch around it is 27 feet wide; what is the length of a ladder that will reach from the out edge of the ditch to the top of the wall? (*S.* 121.) Ans. 45 feet.

9 | 36 4
9 | 36 4
16

9 | 27 3
9 | 27 3
9+16=25

√25=5
5×9=45

34. The roof of a certain building is 32 feet in length is raised in the centre 12 feet; what is the length of the rafters? (*S.* 121.) Ans. 20 ft.

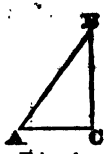
4 | 16
4 | 16

4 | 12 3
4 | 12 3
9+16=25 5×4=20

35. There is a field that is 800 rods long, 600 broad; what is the distance between the opposite corners?
(*S.* 221.) Ans. 1000 rods.

200 | 400=3 200 | 800 4
200 | 400=3 200 | 400 4

9 9+16=25 5+200=1000



Triangle.

36. The hypothenuse of a right angled triangle, A B, is 520 feet, and the base, AC, 312 feet; what is the perpendicular CB? (*T.* 223.) Ans. 416.

A
52 | 520 10 5² 25
52 | 312 4 3² −9

4)16(4×104=416.

37. Required the height of a maypole whose top being broken off, struck the ground at the distance of 15 feet from the foot, and measures 39 feet? Ans. 75 feet.

3 | 39=13² 3 | 15=5²

169 25
−25

12)144(12×3=36+39=75

38. Suppose a pine tree to stand 25 feet from the end of a house 40 feet in length, the foot of the tree being on a level with the foundation of the chimney, which stands in the centre of the house, and a line reaching from the foot of the tree to the top of the chimney be 75 feet; what is the height of the chimney; and if the height of the tree is $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{7}$ of 14 times that of the chimney, what is the length of the line that will reach from the top of the chimney to the top of the tree? (*B.* 195.)

	15	45
15	75 —	—
—	—	3^2
	5^2	—
	9	

25—9=√16=4

Ans. { Length of the line, 75 feet. Height of chimney, 60 " Height of the tree, 120 "

4×15=60

	60
3	1
4	3
7	1
	14
—	120
	—60
	60

15	60	15 45
	4^2	3^2
	—	—
	16	9
	9	
	—	
	√25	5×15=75

39. There is a building 30 feet in length and 22 feet in width, and the eaves project over the wall 1 foot on every side; the roof terminates in a point at the centre of the building, and is there supported by a post, the top of which is 10 feet above the beams on which the rafters rest; what is the distance from the foot of the post to the corners of the eaves, and what is the length of the rafters reaching to the middle of one end; and that of a rafter reaching to one of the ends of the corners of the eaves?

(*A.* 172.) Answers in order: 20 ft., 15,62 ft.; 18,86 + ft., and 22,36 + ft.

40. A hawk perched on the top of a tree 77 feet high, was shot by a sportsman standing off 14 rods; what was the distance in yards that he shot? Ans. 81,15 yds.

(*W.* 149.)

7	77	14 7 $3^2+1^2=10$
	2	33 3 √10=3,16+

§ In working this problem, the square root of 10 is all that it will be necessary to extract.

41. If the diagonal of a rectangular field is 40 rods, and one of the sides 32 rods; what is the other?
(*W.* 140.) Ans. 24.

42. If from a right angled triangle A. B. C., whose base is 12 feet, and perpendicular 16 feet, there be cut off by a line D. E., parallel to the perpendicular, a triangle whose area is 24 square feet; what are the sides of the triangle? Ans. A. D.=10, A. E.=6, D. E.=8.

43. There is a circular pond of 6283½ area in square feet, in the middle of which stood a pole 100 feet high above the surface of the water; now the pole being broken, it was observed to strike the outer edge of the pond; what is the height above the water of the piece that is left standing?

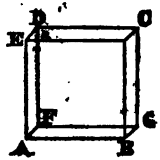
43. A gentleman has in his yard a circular grass plot, that is 25 yards in diameter. Query, what is the length of line that it will take to describe a circle that will contain 9 times the area? (*B.* 88.) Ans. 37½ yds.

45. In a level garden there are two lofty firs, having their tops ornamented with gilt balls; one is 100 feet, the other 80 feet in height, and they are 120 feet apart.—The owner wants to put a fountain in a right line between the trees, to be equally distant from the top of each; what will be the distance from the bottom, and also from the top of each tree?

Ans. { From the top of each ball will be 109,6585 ft., and from the bottom of the higher tree 45ft., and from the lower, 75 ft.

CUBE ROOT.

I. *The cube of any number, is the product of that number multiplied by one of its sides, after the side has been squared.*



A Cube,

Is a figure of 6 equal sides; and extracting the Cube Root consists in finding the length of those sides.

II. *The cube has six sides, which are all equal to each other.*

III. *Cubes whose roots are composite numbers, may be divided by cube factors.*

IV. *Cube numbers whose unit figure is* 5, *may be multiplied by* 8, *and that period reduced to ciphers.*

TABLE OF CUBIC NUMBERS.

NUMBERS	1	2	3	4	5	6	7	8	9	10
CUBES	1	8	27	64	125	216	343	512	729	1000

EXAMPLES.

1. What is the cube root of 6331625? Ans. 185.

	6331625 (R. 198.)		§ First multiply by 8, then as the product ends in 3, the second figure of the root is 7, as found by the table, and we know that the greatest cube of 50 is 3, hence we have as the root 370, which divided by the root of 8 gives the answer.
	8		
	2	370	
3^2	50653000(370		
	27	185	

§ First multiply by 8, then as the product ends in 3, the second figure of the root is 7, as found by the table, and we know that the greatest cube of 50 is 3, hence we have as the root 370, which divided by the root of 8 gives the answer.

2. What is the cube root of 34328125? Ans. 325.
(*B.* 198.)

34328125
8
274625000
8 4 | 1300
2197000(1300 | 325

§ When the sum ends in 5 we can multiply by 8, and as the given sum again ends in 5, we multiply by 8 the second time, and rejecting the ciphers, we have 2197, and looking at the table of cube numbers we find the root to be 1300, and thus we divide by 4 for the answer.

3. What is the cube root of 003,375? Ans. 1,5.
(*B.* 198.)

003375
8

$\sqrt[3]{27000}=30\div 2=15$

4. What is the cube root of 91125? (*R.* 225.) Ans. 45.
5. What is the cube root of 1953125 cubic feet?
(*R.* 226.) Ans.
6. What is the cube root of 941192? Ans. 98.
(*B.* 198.)

| 941192(98

§ By examining the table of cubes, we find that the first figure must be 9, and as the last figure of the given number is 2, if the sum given is a perfect cube the last figure must be 8.

7. What is the cube root of 195112? Ans. 58.
(*P.* 224.)

195112(58

§ Comparing this number with the table, we observe that the first figure cannot be more than 5, and as 2 is the unit figure we must have some figure for the unit that gives 2 when cubed.

8. What is the root of 912673, it being a perfect cube?
Ans.

What is the cube root of the following numbers they being perfect cubes?

1. What is the cube root of 205379? Ans. 59.
2. What is the cube root of 42875? Ans. 35. (*C.* 187.)
3. What is the cube root of 6733740971251? (*T.* 175.) Ans. 8765.
4. What is the cube root of 614125? Ans. 85. (*D.* 173.)
5. What is the cube root of 593191? Ans. 39. (*T.* 225.)
6. What is the cube root of 79507? Ans. 43. (*T.* 225.)
7. Required the cube root of 447697155. Ans. 765. (*B.* 11.)
8. Required the cube root of 253395799552. Ans. 6328. (*S.* 127.)

8	253395799552
64	31674474944
8	3959309368
	494913661(7x1 791.

§ As the given sum ends in two and we find that 8 is the only figure the cube of which gives 2 in the units place, we divide by it; and as the quotient ends in 4, we again look at the table and find that 4 cubed ends in 4; then we divide the quotient by 64, and find as a quotient, 494913661. If we now examine the table, we find that the figure in hundred's place must be 7, and that for units must be 1, and the other will be 9; then we take the root of 64 and 8, and multiply 791 by their product for the answer. $4\times2\times791=6328$ Ans.

TO OBTAIN THE ROOT OF SURDS.

§ 1. *Find by trial a cube near the given number, and call it the supposed cube.*

§ 2. *Then as twice the supposed cube added to the given number is to twice the given number added to the supposed cube, so is the root of the supposed cube to the true or approximate root.*

OR,

§ 3. *The first is to the difference between the first and second, as the third is to the difference between the third and fourth.**

By taking the cube of the root thus found for the supposed cube, and repeating the operation, the root will be had to the greatest degree of exactness.

EXAMPLES.

1. Required the cube root of 28—assign 3 as the root of the nearest cube; then $3\times3\times3=27$, the supposed cube.

Then 27	28	the given number.
2	2	
54	56	
28	27	
82	83	
	3	
82	249(3,0366	true or approximate root.
	246	
	300	
	246	
	540	
	492	
	480	

2. What is the cube root of two? Ans. 1,25+.

Take the cube root of one. $1\times1\times1=1$ | 2 (S. 127.)

2.	2
–	–
2	4
2	1
–	–
4	92(1,3–. Ans.

3. What is the cube root of 7?

§ Take 2 as the cube root. Then as the cube of 2 is 8, we multiply by 8 by 2.

8	7	11 22	1	
2	2		2	
–	–			
16	14		,090909 for correc-	
+7	+8		tion, which must be sub-	
–	–		tracted.	
23	22			

2,000000
90909

1,909091

4. What is the cube root of 66? Ans. 4,0412.

(T. 228.)

§ Take the cube root of the nearest number, which is 64, and the cube root is 4.

64	66	or, 196—194=2		
×2	×2			
–	–	97 194	2	
128	132		4	
+66	+64		–	
–	–		400(0,04124 nearly.	
194	196		388	
	4		–	
			120	
784(4,04124			97	
			–	
			230	
			94	
			–	
			036	

5. Required the cube root of 130.

The nearest perfect cube of a whole number is 125, and the root 5.

```
 125 | 130      or,
  ×2 | ×2                76 380 | 5 0,065+
-----|-----                     | 5
 250 |  260
+130 | +125
-----|-----
76 380 |  385  5,065+
       |  5
```

6. Required the cube root of 123.

```
 125 | 123
  ×2 | ×2
-----|-----          Or,        373 | 2
 250 | 246                          | 5   0,02681+
+123 | 125
-----|-----
 373 | 371
     |   5
     |-----
     | 1855(4,97319
```

§ It is presumed that you have observed, that these corrections added to or subtracted from the assumed root, (as the case may require) will give the cube root very nearly; in fact, as nearly as is necessary in ordinary business. By repeating this operation, the root may be obtained to any degree of exactness.

7. What is the cube root of 8021,6? Ans. 20,105+.

(*C.* 180.) § Suppose 20 to be the cube root, then:

```
$20^3$=8000,0 | 8021,6
            2 |      2
      --------|--------
      16000,0 | 160432
       8021,6 |  80000
      --------|--------
      24021,6 | 240432  120216
       6005,4 | 20  5
      --------|-----------
       3002,7 | 601080(20,016+
```

8. What is the cube root of 3214? Ans. 14,7575+.
(*T.* 129.)

§ First suppose the root is 14 and its cube is equal to $14^3 = 2744$

	2	2
	5488	6428
	3214	2744
4351	8702	9172
	14 7	
	4351	64204,(14,75
		4351
		20694
		17404
		32900

§ As this gives the root very nearly, we may if we wish greater accuracy, take 14,7 and cube it, and then proceed through the same process, and we will get the true root to ten places of decimals.

QUESTIONS FOR EXERCISE.

1. If a bullet 3 inches in diameter weigh 4 lbs.; what would one of the same metal weigh whose diameter is 6 inches? Ans. 32 lbs.

	4
3	2
3	2
3	2
	32

§ As similar figures are to each other as the cubes of their similar sides, or as the cubes of their diameter or circumference, we have 32 for answer.

2. If a solid globe of silver of three inches diameter, is worth $150, what is one worth that measures 6 inches diameter? (*D.* 174.) Ans. $1200.

	150
3	2
3	2
3	2
	1200

3. If a cube of silver whose sides are 2 inches, be worth $20, I demand the length of the side of a cube of like silver, whose value shall be 8 times as much? Ans. 4 in.

	2
	2
	2
x^3	8
x^3	64 $x=4$

4. What is the side of a cubical mound equal to one that is 288 feet long, 216 feet wide, and 48 feet high?

Ans. 144 ft.

	$216,=6^3$	
	$288,6=1728=12^3$	
6	$48\ 8=2^3$	
	144	

§ As 216 is a cube, we take its root, 6; then we multiply 288 by 6, and its product is 1728, and it is a cube; and 48 divided by 6 gives 8, and its root is 2. We then multiply $12\times2\times6=144$, which is much simpler and easier than to multiply the numbers together, and then extract the cube root of the product.

5. If a cube of silver whose sides are 4 inches, be worth £50, what is the side of a cube of like silver, worth 4 times as much? (*B.* 239.) Ans. 6,349+.

6. There is a cube whose diameter is 4 inches; what is the diameter of a cube 4 times as large?

(B. 239.) Ans. 6,928+.

7. Required the cube root of the product of 448×392.

(*T.* 230.) Ans. 56.

8	448 56	$56=8\times7$
8	392 49	$49\times7\times8=$
	$7\times2=14$	

§ Divide 448 and 392 by 8 times 8, which gives as factors 56 and 49; but 56 has as factors 8 and 7, and as 49 is the square of 7, we take $7\times49=$ to the cube of 7; and as 7 is the cube root of 49×7, we have the cube root of $8=2$. As we divided by 8 times 8, take the roots

which are 2×2, and then we have for the

Answer 7×2×2×2=56.

9. A farmer borrowed part of a rick of hay which measured 6 feet every way, and paid back again by 2 equal cubical pieces, each of whose sides was 3 feet.—Query, whether the lender was paid in full?

(*R.* 227.) He was paid only ¼.

10. Mars is about 4000 miles in diameter, the earth 8000; what is their relative magnitudes? Ans. 1 to 8.

(*H. Ast.*)

11. Mercury is about 2000 miles in diameter, and the earth 8000; what is their relative magnitudes?

(*H. Ast.*) Ans. 1 to 64.

The following method of extracting the square root, I find very convenient. Whether it has ever been known and published before, I am unable to say.

1. Extract the square root of 10. Ans. 3,162+.

3×3=9	10	(S. 238.)
3	3	§ Find the nearest root to
—	—	the given number, from the
27	30	table or by trial; then find
+10	+9	the power of that root, and
—	—	multiply that number by 3,
37	39	and also your given number;
—	3	then add the number of which
	—	you have found the root to the
37	117(3,162+	product of the given number,
	111	also add the given number to
	—	the product of the number
	60	taken from the table, then
	37	multiply the product of the
	—	given number and the sum of
	230	the table numbers by the root
	222	of the table number, and di-
	—	vide by the sum of the two
	80	numbers on the left, and the

quotient will be the root very nearly.

GENERAL RULE FOR EXTRACTING THE ROOTS OF ALL POWERS.

I. *At the left of the number whose root is required, arrange as many columns as are equal to the index of the root. Writing* 1 *at the head of the first or left hand column, and zero at the head of each of the others.*

II. *Divide the number into periods of as many figures as the index of the root requires.*

III. *Write the root of the left hand period as the first figure of the true root.*

IV, *Multiply the number in the first column by the root figure, and add the product to the second column ; and the product of this sum by the root figure to the third column, and so proceed, subtracting the product of the last column from the given number.*

V. *Repeat the process, stopping at the last column, and thus proceed, stopping one column sooner each time, until the last sum falls in the second column.*

VI. *To determine the second root figure, consider the number in the column as a trial divisor, and proceed with the second root figure thus obtained, precisely as with the first.*

VII. *Continue the operation until the root is completed or the approximation is carried as far as is desired.*

VIII. *In order to avoid error, observe carefully the value of each root figure, and each product.*

IX. *Thus if the root figure is hundreds, the number in the second column will be hundreds, in the third place ten thousands, in the fourth millions, &c.*

EXAMPLES.

1. What is the square root of 234256 ?

(1) +4 (*c. d.*)	234256	484 Ans.
+4	16	
(1) +8 (*t. d.*)	74,2	
+8	70,4	
(2) +88 (*c. d.*)	3 85,6	
+8	3 85,6	
(2) +96 (*t. d.*)		
+4		
(3) 964 (*c. d.*)		

2. What is the cube root of 122615327,232 ?

1	0	
—	—	. . . .
4	(1) 16 (*c. d.*)	122615327232 | 4968
+4	32	64
—	—	—
8	(1) 48 (*t. d.*)	58615
+4	1161	53649
—	—	—
129	(2) 5961 (*c. d.*)	4966827
+9	1242	4374936
—	—	—
138	(2) 7203 (*t. d.*)	591391232
+9	8856	591391232
—	—	
1476	(3) 729156 (*c. d.*)	
+6	8892	
—	—	
1482	(3) 738048 (*t. d.*)	
+6	11104	
—	—	
14888	(4) 73923904 (*c. d.*)	

PROBLEMS.

The following problems may be useful to Mechanics.

PROBLEM 1.

To find the side of a square equal in area to any given circle.

§ Place 886 and the diameter on the right, and 1000 and the next inferior denomination that corresponds with the answer, on the left.

1. What is the side of a square equal in area to a circle of 3 feet diameter ? Ans. 2,6 58 ft.

1000	886
	3
	—
	2658

2. What is the side of a square equal in extent to a circle of 40 inches? Ans. 2,953½ ft.

1000	886
3 12	40
	2,953½

PROBLEM 2.

To find the side of the greatest square that can be inscribed in a given circle.

§ Place 707 and the diameter of the circle on the right; and on the left place 1000 and the next inferior denomination that corresponds with the answer.

1. What is the side of the greatest inscribed square that can be drawn in a circle of 50 inches diameter? Ans. 2,9458 ft.

50 1000	707 29458
12	50
	2,9458

2. What is the side of the greatest square that can be inscribed in a circle of 960 inches? Ans. 56,56 ft.

PROBLEM 3.

To find the length of one of the sides of the greatest cube that can be cut from a globe of a given diameter.

§ Place 577, with the diameter, on the right; and place 1000 and the next inferior denomination that corresponds with the answer, on the left.

1. What is the length of the greatest cube that can be cut from a globe of 90 inches? Ans. 51,93 inches, or 4,32¾ feet.

1000	577
	90
	51,93

1000	577
4 12	90 3
	1731
	4,3275

2. What is the length of the side of the greatest cube that can be cut from a globe of 82 inches diameter?

Ans. 47,314 in., or 3,9360 ft.

1000	577	1000	577 96
	82	6 12	82 41
	1154		96
	4616		384
	47,314		3,936

PROBLEM 4.

To find the length of the side of the greatest equilateral triangle that can be inscribed in a given circle.

§ Place 87 and the diameter on the right, and 100 and the next inferior denomination that corresponds with the answer, on the left:

1. What is the length of the side of the greatest equilateral triangle that can be inscribed in a circle of 54 inches diameter?

Ans. { 46,98 inches, or 3,915 feet.

100	87	100	87
	54	2 12	84 9
	3,48		783
	43,5		3,915
	46,98		

2. What is the length of the side of the greatest equilateral triangle that can be drawn in a circle of 60 inches diameter.

Ans. 52,2 inches, or 4,35 feet.

3. What is the least diameter of a circle that will circumscribe an equilateral triangle whose sides are 6½ inches?

Ans. 7,47+ inches.

87	6,5
	100
	7,47

PROBLEM 5.

To find the length of the side of the greatest figure that can be inscribed in a given circle.

§ 1. If it is a *pentagon* or 5 sided figure, place 587 and the diameter on the right, and 1000 and the next inferior denomination on the left.

§ 2. If it is a *hexagon* or 6 sided figure, place 5 on the right with the diameter, and 10 and the next inferior denomination on the left.

§ 3. If a *heptagon*, place 437 and the diameter on the right, and on the left place 1000 and the next inferior denomination.

§ 4. If it is an *octagon* place 383 and the diameter on the right, and on the left place 1000 and the next inferior denomination.

§ 5. If it is a *nonagon*, place 337 and the diameter on the right, and place on the left the numbers as before.

§ 6. For a *decagon*, place 310 and the diameter on the right, and place on the left 1000 and the next inferior denomination that corresponds with the answer.

NUMBERS.

For a square	4	sides	take 707
Triangle	3	"	" 870
Pentagon	5	"	" 587
Hexagon	6	"	" 500
Heptagon	7	"	" 437
Octagon	8	"	" 383
Nonagon	9	"	" 337
Decagon	10	"	" 310
Undecagon	11	"	" 282
Dodecagon	12	"	" 260

1. What is the length of the side of the greatest hexagon that can be inscribed in a circle of 54 inches diameter?

2 1000 | 500
54

27

Ans. { 27 inches, or 2 ft. 3 in.

PROBLEM 6.

To find the area of a regular polygon when the sides are known.

§ Place the square of the side of the polygon and the number that stands opposite the number of sides in the table on the right, and on the left place the next inferior denomination that corresponds with the answer.

TABLE.

No. of sides.	*Names.*	*Multipliers.*
3	Trigon	0,433013
4	Tetragon	1,000000
5	Pentagon	1,720477
6	Hexagon	2,598076
7	Heptagon	3,633912
8	Octagon	4,828427
9	Nonagon	6,181824
10	Decagon	7,694209
11	Undecagon	9,365640
12	Dodecagon	11,196152

1. The side of a hexagon is 5 feet 4 inches; what is the area?

3 12 | 64
3 12 | 64
2,598

73,899 feet.

2. Required the area of an octagon whose side is 16 feet.
Ans. 1236+ feet.

PROBLEM 7.

To find the solidity of a cunus or wedge.

§ Add twice the length of the base to the length of the edge, and place the sum on the right with the length and breadth of the base, and on the left place 6 and the next inferior denomination that corresponds with the answer.

1. How many solid feet in a wedge whose base is 5 feet 4 inches long and 9 inches broad, the length of the edge

being 3 feet 6 inches, and the perpendicular height 2 feet 4 inches? Ans. 4,1319 ft.

12	64×2+42
12	28
6	9
12	
	4,1319

PROBLEM 8.

To find the solidity of the segment of a sphere.

§ Square the radius and multiply it by 3, and to this add the square of the height; place this sum on the right, with 3,1416, and on the left place 6 and the next inferior denomination that corresponds with the answer.

1. What is the solidity of a segment of a sphere, the diameter of whose base is 24 inches and its height 9 inches? Ans. 1,4 ft. nearly.

12	$\overbrace{12\times12\times3}+\overbrace{9\times9}$
12	
6	3,1416
12	
	1,398

PROBLEM 9.

To find the greatest diameter of three equal circles, that can be inscribed in a given circle.

§ Place 464, and the diameter of the greater circle on the right; and place 1000, with the next inferior denomination that corresponds with the answer, on the left.

1. What is the greatest diameter of each of the three circles that can be inscribed in a circle of 100 inches diameter? Ans. 46,4 in.

10 1000	464
	100
	46,4

2. What is the greatest diameter of each of the three circles that can be inscribed in a circle of 25 inches.

Ans. 11,6 in.

40 1000 | 464
25
11,6

PROBLEM 10.

To find the greatest diameter of four equal circles that can be inscribed in a given circle.

§ Place 416 and the given diameter, on the right; and place 1000 and the next inferior denomination that corresponds with the answer, on the left.

1. What is the greatest diameter of each of four circles, that can be inscribed in a circle of 22 inches diameter?

Ans. 9,1520 in.

1000 | 416
22
9,152

PROBLEM 11.

To find the area of an elipse.

§ Place the transverse and conjugate diameters, with 7855, on the right; and place 10000, together with the next inferior denomination that corresponds with the answer, on the left.

1. What is the area of an elipse whose conjugate diameter is 10 inches, and its transverse 12.

Ans. 94,260 in.

12
10000 | 10
7855
94,260

PROBLEM 12.

To find the weight of an iron ball from its diameter or circumference.

§ Place the cube of the diameter and 56 on the right, and on the left place 4 and 100.

1. What is the weight of an iron ball of 6 inches diameter? Ans. 30¼ lbs.

	6
4	6 3
	6 3
100	56
—	—
	30,24

2. What is the diameter of a cast iron ball that weighs 9 pounds? Ans. 4 in., nearly.

3. What is the weight of a cast iron ball that measures 5,54 inches in diameter? Ans. 24 lbs., nearly.

4. What is the diameter of a ball of cast iron that weighs 75 pounds? Ans. 10+ in.

PROBLEM 13.

To find the weight of a leaden ball from its diameter or circumference.

§ Place the cube of the diameter and 215 on the right, and 1000 on the left.

1. What is the weight of a leaden ball that is 6,6 inches in diameter? Ans. 88 lbs.

PROBLEM 14.

The surface of all similar bodies are to each other as the squares of their like dimensions, (such as diameters circumferences, like linear sides, &c.,) and their solidities as the cubes of those dimensions.

1. The diameter of the earth is 7920 miles, and that of the moon 2160 miles. Required the ratios of their surfaces and their solidities? Ans. { sol., 1 to 49. sur., 1 to 13¼.

PROBLEM 15.

To find the surface of a cylindrical ring.

§ Place the sum of the rings, diameters, the inner diameter of the circle, and the number 987, on the right; and 100 and the next inferior denomination that corresponds with the answer, on the left.

1. The thickness of a cylindrical ring is 4 inches, and the diameter of the circle 16 inches; what is its convex surface? Ans. 789,6 in.

	20
100	4
	987
	789,6

To find the solidity.

§ Place the sum of the rings, diameters, the square of the thickness, and 989, on the right; and place 100 and 4, on the left.

1. What is the solidity of an anchor ring, whose inner diameter is 7 inches, and thickness in metal 3 inches? Ans. 222,5 in.

	10
	3
4	3
100	989
	8901
	222,5

PROBLEM 16.

To find the solid contents of a circular elipse, or gothic vaulted roof.

§ Find the area of one end; that multiplied by the length, will give the solidity required.

1. What is the solid contents of a semi-circular vault, whose span is 120 feet, and diameter 40 feet?

Ans. 75398,4 ft.

	40 2
	40
2	120
10000	7854
	62832
	125664
	75398,4

2. Required the solidity of an eliptical vault, whose span is 40 feet, height 12 feet, and length 80 feet?

Ans. 301593,6 cubic ft.
1117,02 cubic yd.

	40	3	40
	40	3	40
	12	3	12
4	80	4	7854
10000	7854	10000	80
	301593		

PROBLEM 16.

To find the solid contents of a dome.

1. What is the solid contents of a circular dome, the diameter of whose base is 60 feet, and height 60 feet?

Ans. 69,8 cubic yd., nearly.

3	60 2	
3	60 2	§ Find the area of the base, and multiply that by $\frac{2}{3}$ of the height.
3	7854	2618
25 10000	2	
3		
75	5236	(69,8

2. In a hexagonical spherical dome, one side of the base is 20 feet; what is the solidity?

Ans. 444,14+ cubic yd.

PROBLEM 17.

To find the superficial contents of a spherical dome.

1. What will the painting of a hexagonical spherical dome cost, at 12½ cts. per yard, that measures 20 feet on each side of the base? Ans. $28,86.

10000	25980,
3	20
3	20
8	2
	1

28,86

PROBLEM 18.

To find the diameter of cog wheels.

EXAMPLES.

1. What is the diameter of a bevel wheel of 150 cogs, the pitch being 1¼ inches? Ans. 5 feet.

2 100	150	5
3	8	
4	5	

§ Place on the right the number of cogs, and 8 and the pitch or distance from center to center of cogs; and on the left place 100 and 3.

5 Ans.

2. What is the diameter of a bevel wheel of 150 cogs, 2 inch pitch? Ans. 8 feet.

3. What is the diameter of a wheel of 130 cogs, distance from center to center of cog 2¼ inch? Ans. 7,8 feet.

4. What is the diameter of a cog wheel of 230 cogs, pitch 2½ inches? Ans. 13⅓ feet.

MISCELLANEOUS EXAMPLES.

1. A snail was observed to climb up a pole that was 20 feet high, each day 8 feet, but descended 4 feet every night; in what time did he get to the top of the pole? (*D.* 266.) Ans. 4 days.

2. A sloth was observed to ascend a tree at the rate of $9\frac{3}{4}$ inches each day, but to descend $3\frac{2}{3}$ inches each night; at this rate, in what time would it reach the top of a tree 100 feet high? (*C.* 225.) Ans. $196\frac{4}{7}\frac{8}{3}$ days.

A NEW PROBLEM.

3. A gentleman having a silver globe, found that when it was filled with wine that cost $3 per gallon, it was worth the same as the globe valued at $12\frac{1}{2}$ cents per superficial inch; what was the diameter of the globe?

(*S. C.*) Ans. $57\frac{3}{4}$ in.

(For solution see the Globe.)

x^2	x^2
7 8 5 4	7 8 5 4
4	4
. . .	6 2
4 8	1
3	231
x	$57\frac{3}{4}$

4. A pit 60 feet diameter and 18 feet deep, is to be filled; and three other pits, each of the same depth, are to be dug, but only 20 feet diameter. Query, is the dirt from the three sufficient to fill the large pit?

(*S. C.*) They will fill only $\frac{1}{3}$ of it.

5. Sold a watch for $24, and by so doing I gain as much per cent. as the watch cost me; what was the cost of the watch?

6. What would be the produce of a kernel of wheat in 11 years, at 20 fold the produce of each year, being sown the next; allowing 5000 kernels to a quart?

(*W.* 166.) Ans. 64,000,000 bu.

7. What is the weight of the pea of a steel yard which at 39 inches from the centre of motion will balance a hog weighing 208 lbs., suspended at the draught, and $\frac{1}{4}$ of an inch from the centre of motion? Ans. 4 lbs.

8. How far may a mountain be seen that is $1\frac{1}{2}$ miles high, from the top of a ship's mast 50 feet in height?

Ans. $117\frac{2}{3}$ miles.

$\sqrt{(d.\ \text{earth}+5)\times 50}=8\frac{2}{3}$ miles.
$\sqrt{d.\ \text{earth}+7920\times 7920}=109$ miles.

9. Sailing up the Chesapeake Bay, I saw the Baltimore Light House just in the horizon; what was the height of the Light House, my eye being 6 feet above the water, and the distance to the Light House $16\frac{1}{2}$ miles?

Ans. $121\frac{1}{2}$ ft.

2	27
2	27
3	2
	$121\frac{1}{2}$

10. Just rising from the sea I've seen
When standing in the shrouds,
The lofty peak of Teneriffe
That penetrates the clouds.
Just 50 leagues from it was I,
My reckoning being true,
And from the water to my eye
The feet were eighty-two;
Suppose these observations just,
To make the question brief,
Above the level of the sea
How high was Teneriffe?

Ans. { 2,43 miles, or 2,422+.

§ The only apology I offer for inserting the following questions, is, that they are so frequently presented for solution, that it will save time to present them already solved.

In almost all questions of this kind, the proportions given are incorrect. In the example: Two men were employed to dig 100 yards of ditch for $100; A. is to receive $\frac{1}{2}$ of it and B. the other $\frac{1}{2}$. Now B. agrees to dig at one end, and A.

at the other, B. receiving $1,25 per rod. and A. $0,75.—Query, how many rods has each to dig to receive his $50?

§ This sum cannot be worked, and for this reason; that subtracting 25 from 100 decreases 100 more in proportion than adding 25 ts 100 increases it; hence, it destroys the proportion. We find that 40 rods at $1,25 per rod, equals $50,00, and 60 rods at $75 per rod, equals $45; from which it appears that 100 rods are dug for $95.

These questions belong to Algebra; but from the Algebraic solution an arbitrary rule may be formed for solving them, when the correct statement is given. Two young men purchase 300 acres of land for $600, each paying the same amount, and agreed to divide according to the quantity. A. has his choice, and pays 75 cts. per acre more than B; the question is, to find how many acres each received, and the price per acre.

x=B's price per acre,

$x+75$=A's " " "

$\frac{300,00}{x}$=B's acres of land,

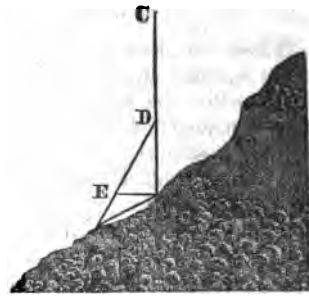
$\frac{300,00}{x+75}$=A's " " "

$\frac{300,00}{x}+\frac{300,00}{x+75}$	300	
by 300 $\frac{100}{x}+\frac{100}{x+75}$	1	Divide through Then clear of fractions.
and transposing $\frac{x^2-125x}{}$	7500	Then completing the square and extracting
the root and we have x	1,693	A's price=$1,693 B's " =$2,443 B's acres=177,2 A's " =122,8

§ The following questions are inserted for those who have the ability and leisure to work them.

1. What weight must be attached to the long arm of a lever, to keep down the safety-valve of a hydrostatic oil press, the length of lever being 30 inches, (long arm 27 inches, short arm 3 inches,) diameter of safety-valve $\frac{1}{2}$ inch. Upon the piston of the press (which is 12 inches diameter,) there is to be exerted a pressure of 300 tons?

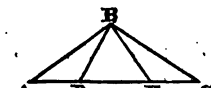
Ans. $57\frac{47}{54}$ lbs.



5. On the side of a mountain grew a lofty fir, which being broken by the wind but not severed, the top C. struck below the foot of the tree B. 50 feet, and a right line B. E. from the base of the tree to the body was 30 feet; what was the length of the pieces, the height of the tree B. C. being 150 feet?

2. At what distance must a weight $6\frac{3}{4}$ lbs., be placed to balance 1 ton on a platform weighing scale; the length of the platform levers being 14 feet, short arms $\frac{1}{20}$ the length of long arms, and the fulcrum of weighing lever being $2\frac{3}{8}$ inches from the draught?

3. If $7\frac{1}{2}$ oxen eat $4\frac{1}{2}$ acres of grass in 2 weeks, and 12 oxen eat up $4\frac{2}{5}$ acres in 5 weeks, how many oxen will eat 60 acres in 8 weeks, the grass being at first equal on every acre, and growing uniformly?



6. In two right angled triangles, A. B. E. and D. B. C., right angled at B., we have the side A. B.=40 inches, B. C.=50 inches, and D. E.=30 inches; required D. B. and B. E.

4. If $5\frac{1}{2}$ oxen eat $7\frac{1}{2}$ acres of grass in $2\frac{1}{4}$ weeks, and $15\frac{1}{3}$ oxen eat up $25\frac{1}{2}$ acres in $8\frac{1}{3}$ weeks, how many oxen will eat up 36 acres and its growth in 20 weeks, the grass growing uniformly in the first and second cases, and $\frac{4}{5}$ as fast in the last?

THE

PRUSSIAN CALCULATOR.

KEY.

As it is necessary to have a quick apprehension of those numbers which are divisible by others, the following observations are made :

§ 1. Every number that cannot be divided by any other number greater than one, is a prime number.

§ 2. Two or more numbers that have no common divisor, are said to be prime to each other.

§ 3. Every prime number is prime to all other numbers, except its own multiples.

§ 4. We cannot determine at once in regard to large numbers, whether they are prime or not; but the following properties of numbers, will enable us to determine all the divisors of any number :

1. Two is a factor of any number that ends in 0, 2, 4, 6 or 8. For as 2 will divide 10, so it will divide any number of tens plus 2, 4, 6, 8,

2. All numbers that can be divided by 2, are called even numbers ; all others are called odd numbers.

3. All numbers terminating in 5 or 0, can be divided by 5. For as 5 will divide 10, so it will divide any number of tens, plus 5.

4. Three or 9, is a factor of all numbers in which the sum of the figures are exactly divisible by 3 or 9. For if from 10, 100, 1000 we take one, they are then equal to 9, 99, and 999; and as the sum of each of these numbers is divisible by 3 or 9, so any other number can be divided by 3 or 9 that possesses the same properties.

5. Let us examine the number 34794. If we add 3+4+7+9+4=27, we find the sum is 27; and as 27 can be divided by 3 or 9, so can 34794.

6. Eleven is a factor of all numbers in which the sum of the odd digets, (the 1st, 3d, 5th, 7th, 9th, &c.,) and the sum of the even digets, (the 2d, 4th, 6th, 8th, &c.,) are equal to their difference, or some multiple of 11. For any number of tens, hundreds, thousands, &c., (which represent the even digets) increased by as many units, will be divisible by 11; also, any number of hundreds, thousands, ten-thousands, (which represent the odd digets) diminished by as many units, will also be divisible by 11. Take the number 635173, and add 6 to 600000, and then it is divisible by 11; also, take 30000 and subtract 3, and the remainder is divisible by 11. 5000+5 is also divisible, and 100+10, 70+7 are divisible by 11, so 3—3=0. Hence, 635193 is also divisible by 11.

7. Four is a factor of all numbers the two terminating figures of which can be divided by 4.

8. Twenty-five is a factor of all numbers in which the terminating figures are 25, 50, 75, or two or more zeros.

9. Every number that is divisible by two or more numbers that are prime to each, is divisible by their product.

10. Every even number that is divisible by 3, is also divisible by 6, and every even number that is divisible by 9, is also divisible by 18.

11. Every number divisible by 3 or 9, in which the two terminating figures can be divided by 4, is also divisible by 12 or 36.

12. Every number divisible by 3 or 9, whose terminating figure is 0 or 5, is also divisible by 15 or 45.

13. Every prime number greater than 2, is also one greater or one less than some multiple of 4.

14. Every prime number that has no prime factor equal to or less than its square root, is itself a prime number.

MULTIPLICATION OF NUMBERS

CONTRACTIONS.

1. When the multiplier consists of any number of nines, increase by one and subtract the multiplicand from the product.

Thus, multiply 3884 by 9999.

38,840,000
3,884

38,836,116

2. To multiply by 5, divide the multiplicand by ,2.

3. To multiply by 25, divide by ,04.

4. To multiply by 75, multiply by 100 and subtract one-fourth.

5. To multiply by 125, divide the multiplicand by ,008.

6. To multiply by 375, divide by ,008, and multiply the quotient by 3.

7. To multiply by 625, divide the multiplicand by ,4 and that quotient by ,04.

8. To multiply by 875, multiply by 1000, and subtract one-eighth of the product.

9. To square any number ending in 5, multiply the number of tens by one more than itself, and place 25 at the right of the product.

10. When the tens in any two numbers are alike, and the sum of the units is ten, multiply the number of tens by one more than itself for the hundreds, and place the product of the units at the right, for tens and units.

11. The sum of two numbers multiplied by their difference, is equal to the difference of their square. Hence, we may readily find the product of two numbers, the one of which is as much above, as the other is under a certain number of tens.

53 by 47 $50+3$
$50-3$

$2500-9=2491$

12. To square any number between 50 and 60, add the units in the given number to 25 for the hundredths, and annex the square of the units at the right for the tens and units.

SIGNS

|	Sign of Division or of Equality.
÷	Sign of Division.
=	Sign of Equality.
+	Sign of Addition, as 8+4=12; or, it signifies when placed after the answer, that there is still a remainder.
—	Signifies less, as 8—4=4, or if placed after the answer, it shows that the answer is too great.
⅟	Sign of Division.
×	Sign of Multiplication.
√	Sign of the Roots, as $\sqrt{16}=4$, shows that the square root of 16 is equal to 4; or, $\sqrt[3]{27}=3$, shows that the cube root of 27 is equal to 3.
$6\times\overbrace{2+3}$	Sign that 6 is to be multiplied by 2 after 2 has been added to 3, as 5×6=30.
$6\times\overbrace{5-2}=18$	Shows that 6 is to be multiplied by 5 after 2 has been taken from 5.
24 | $2\times\overbrace{6+8}$ / 12 — 14 Ans.	Shows that 2 is to be multiplied by 6 after 8 has been added to 6, and that product by 12, and the last product is to be divided by 24; or, which is the same, cancel 12 and 2 on the right, and 24 on the left, then add 6 and 8 together.
$4^2=16$	Signifies that the square of 4=16.
$16^{\frac{1}{2}}=4$	Signifies that the square root of 16 is 4.
x^3 x^5	Whole figures at the top and right, signify the powers to which those numbers are to be raised.

x^4 x^4 Fractions at the top and right, designate the power to which the number has been raised, or that the roots of those numbers are to be extracted.

40
400 When all the figures at the right of ciphers are canceled and the ciphers stand uncanceled, they have the same value as though 1 stood at the left.

2 | 1
3 | 2
1 | 3 When all the figures are canceled, one is always understood as the answer.

TABLES.

I.

Table for ascertaining the number of days, from any day in the year to any other day.

1st	mo.,	Jan.	0	5th	mo.,	May	120	9th mo., Sept. 243
2d	"	Feb.	31	6th	"	June	151	10th " Oct. 273
3d	"	Mar.	59	7th	"	July	181	11th " Nov. 304
4th	"	Apr.	90	8th	"	Aug.	212	12th " Dec. 334

RULE.

§ To the given day of each month add the tabular number for the month and subtract the less from the greater.

§ If the two dates are in different years, subtract the result thus found from 365.

§ In leap years, add one to the number after the 28 of February.

TABLES OF MONEY.

II.

UNITED STATES.

10 mills	-	=1 cent	ct.
10 cents	-	=1 dime	d.
10 dimes	-	=1 dollar	$
10 dollars	-	=1 eagle	E.

GREAT BRITAIN.

4 farthings (qrs.)	-	=1 penny	d.
12 pence	-	=1 shilling	s.
20 shillings	-	=1 pound	£
21 shillings	-	=1 guinea	G.

III.

EXCHANGE AND REDUCTION OF CURRENCY.

This branch of Arithmetic is of little or no use to men in common business, with the exception of what relates to the value of coins in circulation in the different States. In almost all the States accounts are now kept in "Federal Money."

In New England and Virginia 6 shillings make the dollar.

In New York and North Carolina 8 shillings to the dollar.

In New Jersey, Pennsylvania, Delaware and Maryland 7 shillings and 6 pence, and in South Carolina 4 shillings and 8 pence.

One dollar is of the same value in all the States.

TABLE OF PROPORTIONS OF THE DIFFERENT CURRENCIES.

In New England and Virginia,	£3=$10 3s= ,50 18d= ,25
In New York and North Carolina,	£2= $5 4s= ,50 24d= ,25

In New Jersey, Pennsylvania, Delaware and Maryland,	£3= $8 3s= ,40 9d= ,10
In South Carolina and Georgia,	£7=$30 7s=1,50 14d= ,25

W

IV.

AVOIRDUPOIS WEIGHT.

10 grains (gr.)	=1 scruple	sc.
3 scruples	=1 drachm	dr.
16 Drachms	=1 ounce	oz.
16 ounces	=1 pound	lb.
25 pounds	=1 quarter	qr.
4 quarters	=1 hundred weight	cwt.

☞ 28 pounds is called a quarter in some of the Arithmetics, or 112 pounds=one hundred weight; but in business, 25 is the standard qr.

V.

TROY WEIGHT.

24 grains (gr.)	=1 pennyweight	pwt.
20 pennyweights	=1 ounce,	oz.
12 ounces	=1 pound,	lb.

VI.

APOTHECARIES' WEIGHT.

24 grains (gr.)	=1 scruple	℈
3 scruples	=1 drachm	ʒ
8 drachms	=1 ounce	℥
12 ounces	=1 pound	lb.

VII.

TABLES OF MEASURES.

LONG MEASURE.

3 barley corns (bc.)	- -	=1 inch	in.
12 inches	- - -	=1 foot	ft.
3 feet	- - -	=1 yard	yd.
5½ or 5,5 yards	- - -	=1 rod or pole	r. or p.
40 rods	- - - -	=1 furlong	fur.
8 furlongs	- - -	=1 mile	m.
3 miles	- - -	=1 league	lea.

§ A palm is 3 inches; a hand is 4 inches; a span is 9 inches; a pace is 3 feet; a fathom is 6 feet; a knot is a geographical mile. 69½ miles or 60 geographical miles = 1 degree, and 360 degrees measures the earth's circumference.

VIII.

CHAIN MEASURE.

7,92 inches	- - -	=1 link	l.
25 links	- - -	=1 pole	p.
4 poles	- - -	=1 chain	ch.
10 chains	- -	=1 furlong	fur.
8 furlongs	- -	=1 mile	m.

A lot of land measuring 10 chains in length and one in breadth is an acre.

IX.

SQUARE MEASURE.

144 square inches	- -	=1 square foot	ft.
9 square feet	-	=1 square yard	yd.
30¼ or 30,25 square yards		=1 square rod	rd.
40 square rods		=1 square rood,	ro.
4 square roods, or 10 square chains	-	=1 acre	a.
640 square acres	-	=1 square mile	m.

X.

CUBE MEASURE.

1728 cubic inches - -	=1 cubic foot	c. f.
40 feet of round timber, or 50 feet of hewn timber	=1 ton	T.
16 cubic feet - -	=1 cord foot	c. ft.
8 cord feet - -	=1 cord	
42 cubic feet - -	=1 ton of shipping.	
4 feet wide 4 feet high and 8 feet long	=1 cord of wood	

XI.

CLOTH MEASURE.

2¼ or 2,25 inches - -	=1 nail	n.
4 nails - - -	=1 quarter	qr.
4 quarters - - -	=1 yard	yd.

XII.

ASTRONOMICAL OR CIRCULAR MEASURE.

60 seconds (″) - -	=1 minute	m. or ′
60 minutes - -	=1 degree	°
30 degrees - -	=1 sign of the Zodiac.	
12 signs - -	= the orbit of a planet.	

In every circle there are 360 degress.

XIII.

COMMON LIQUID MEASURE.

4 gills (gl.) - - -	=1 pint	pt.
2 pints - - -	=1 quart	qt.
4 quarts - - -	=1 gallon	gal.

XIV.

DRY MEASURE.

2 pints (pts.)	=1 quart	qt.
8 quarts	=1 peck	pk.
4 pecks	=1 bushel	bu.
36 bushels	=1 chaldron	chal.

XV.

TIME.

60 seconds (sec.)	=1 minute	min.
60 minutes	=1 hour	h.
24 hours	=1 day	dy.
7 days	=1 week	w.
30 days	=1 month	m.

TABLES OF ALIQUOT PARTS.

Of a dol. cts. $	*Of a £.* s. d. £.	*Of a shil.* d. s.	*Of a ton.* cwt. qr. ton.	*Of a cwt.* qr. lb. cwt.	*Of a year.* mo. d. y.
50 = ½	10 = ½	6 = ½	10 = ½	2 = ½	6 = ½
33⅓ = ⅓	6 8 = ⅓	4 = ⅓	5 = ¼	1 = ¼	4 = ⅓
25 = ¼	5 = ¼	3 = ¼	4 = ⅕	16 = ⅛	3 = ¼
20 = ⅕	4 = ⅕	2 = ⅙	2 2 = ⅛	14 = ⅛	2 12 = ⅕
16⅔ = ⅙	3 4 = ⅙	1½ = ⅛	2 = 1/10	8 = 1/14	2 = ⅙
12½ = ⅛	2 6 = ⅛	1 = 1/12	1 1 = 1/16	7 = 1/16	1 15 = ⅛
10 = 1/10	2 = 1/10	¾ ¾ = 1/16	1 = 1/20	4 = 1/28	1 10 = ⅙
8⅓ = 1/12	1 = 1/20	½ = 1/24	2 = 1/40	2 = 1/56	1 6 = 1/10
6¼ = 1/16	10 = 1/24	¼ = 1/48	8 = ⅛	1 4 = ⅛	4 24 = ⅛
5 = 1/20	8 = 1/30	9 = ¾	12 = ⅛	1 20 = ⅛	7 6 = ⅛
4 = 1/25	4 = 1/60	8 = ⅔	16 = ⅘	2 8 = ⅘	9 18 = ⅘
3⅓ = 1/30	3 = 1/80	7½ = ⅝	15 = ¾	2 24 = ⅛	4 15 = ⅜
2 = 1/50	2 = 1/120	4½ = ⅜	7 2 = ⅜	3 12 = ⅛	7 15 = ⅛

MISCELLANEOUS TABLE.

12 things	=1 dozen.
12 dozen	=1 gross.
12 gross	=1 great gross.

20 things	=1 score.
5 scores	=1 hundred.
6 scores	=1 longhundred.
24 sheets	=1 quire.
20 quires	=1 ream.

A sheet folded into

2 leaves	= a folio fol.
4 leaves	= a quarto 4 to.
8 leaves	= an octavo 8 vo.
12 leaves	= a duodecimo 12 mo.
16 leaves	= 16 mo.
18 leaves	= 18 mo.
24 leaves	= 24 mo.

A barrel of flour	=196 lbs.
A barrel of pork or beef	=200 lbs.
A barrel of shad or salmon	=200 lbs.
A barrel of salt	=280 lbs.
A barrel of butter	=224 lbs.
A barrel of soap	=256 lbs.
A furkin of butter	= 56 lbs.
A bushel of barley	= 48 lbs.
A bushel of rye	= 56 lbs.
A bushel of wheat	= 60 lbs.
A bushel of oats	= 32 lbs.
A bushel of flour or salt	= 56 lbs.
A bag of rice	=168 lbs.
A chest of tea	= 84 lbs.
A gallon of tarnoil	= 7½ lbs.
A gallon of vinegar	= 8 lbs.
A gallon of molasses	= 11 lbs.
A stone of iron or shot	= 14 lbs.
A stone of meat	= 5 lbs.
A stone of glass	= 8 lbs.
A stone of cheese	= 16 lbs.
A stone of hemp	= 32 lbs.
A faggot of steel	=120 lbs.
A seam of glass	=120 lbs.

ANSWERS TO THE SUMS

That have not the Answers given in the body of the Work.

2	example.	Ans.	18	Page	8
3	"	"	$7\frac{1}{2}$	"	8
11	"	"	$\frac{2}{3}$	"	10
12	"	"	$\frac{1}{3}$	"	10
13	"	"	$\frac{1}{3}$	"	10
14	"	"	20	"	10
15	"	"	2	"	10
16	"	"	588	"	10
17	"	"	10	"	10
18	"	"	$\frac{11}{16}$	"	11
19	"	"	$3\frac{1}{3}$	"	11
20	"	"	$10\frac{2}{3}$	"	11
21	"	"	1	"	11
22	"	"	$5\frac{1}{2}$	"	11
23	"	"	$37\frac{1}{2}$	"	11
24	"	"	45	"	11
25	"	"	$\frac{27}{50}$	"	11
26	"	"	$2\frac{1}{2}$	"	11
27	"	"	2000	"	11
28	"	"	8	"	12
29	"	"	$\frac{1}{2}$	"	12
30	"	"	2	"	12
31	"	"	1	"	12
32	"	"	$2\frac{3}{10}$	"	12
5	"	"	$\frac{4}{5}$	"	13
6	"	"	5	"	13
7	"	"	$\frac{1}{12}$	"	13
8	"	"	$\frac{1}{3}$	"	13

Example,		Answer.		Page.
9	"	$\frac{1}{2}$	"	13
4	"	$\frac{1}{16}$	"	14
17	"	$4, $8, $12, $16, $20, $24, $28, $32, $36, $40, $44, $48, $52, $56, $60,	"	16
18	"	$22, $44, $21,78,	"	16
19	"	$180, $150, $120, $240, $360, $420, $480,	"	16
20	"	$6, $9, $15, $18, $21, $24, $27, $30, $33, $36, $39,	"	16
16	"	A has $\frac{1}{5}$ the most,	"	18
20	"	190 bbls.	"	18
18	"	Answer correct, $4 and $5,	"	23
19	"	correct, $14,94.	"	23
27	"	Answer correct, $6,	"	25
7	"	" " $120.	"	35
2	case 3,	" $\frac{1}{3}$	"	15
5	"	" $7\frac{1}{2}$	"	43
2	"	" $2,60\frac{1}{3}$	"	48
3	" correct,	" ,48$\frac{1}{3}$	"	"
4	" correct,	" $5,19\frac{3}{13}$	"	"
15	"	" 80 acres,	"	51
4	" correct,	" 339,2928 cubic feet	"	62
12	"	" $15	"	68
8	"	" 5 tons,	"	71
2	"	" 600 revolutions,	"	77
2	" case 2	" 4523,9040	"	76
2	" "	" 201,0624	"	"
3	" "	"	"	77
4	"	" 25,1328 tons	"	78
2	"	" 36960 lbs.	"	78

Example.				Answer.		Page.
3	"		"	20790 lbs.	"	78
4	"		"	9240 lbs. 14437$\frac{1}{2}$ lbs. 18857$\frac{1}{7}$ lbs. 46777$\frac{1}{2}$	"	"
2	"		"	8000 lbs.	"	79
3	"		"	$4\frac{1}{18}$ tons,	"	79
2	"		"	28125 lbs.	"	80
3	"		"	8000 lbs.	"	"
4	"		"	54000 lbs.	"	"
5	"		"	24000 lbs.	"	"
2	case 4	correct	"	$15\frac{5}{6}$	"	105
6	"	correct	"	$1\frac{51}{52}$	"	"
7	"	correct	"	17	"	"
2	"	correct	"	$\frac{3}{14}$	"	107
8	"		"	$\frac{1}{32}$ or 1	"	117
9	"		"	$\frac{1}{882}$ or $\frac{3}{6}$	"	"
29	"		"	15 days,	"	119
30	"		"	740 men,	"	119
31	"		"	$50\frac{294335}{687076}$ yds.	"	119
32	"		"	$297\frac{5}{6}$ days,	"	"
5	"		"	$20	"	173

ERRATA.

38 page, 3d example, in stating, read 180,60.

48 " 4th " 75 cts. in place of 65.

109 " 4th " read what fraction of a guinea is $\frac{4}{5}$ of a £.

INDEX.

PART I.

PART II.

PART III.

APPENDIX.

KEY.